# 国際共同研究 Niagara Project と Matrix Art

津山高専　　一般・数学　　有本　茂

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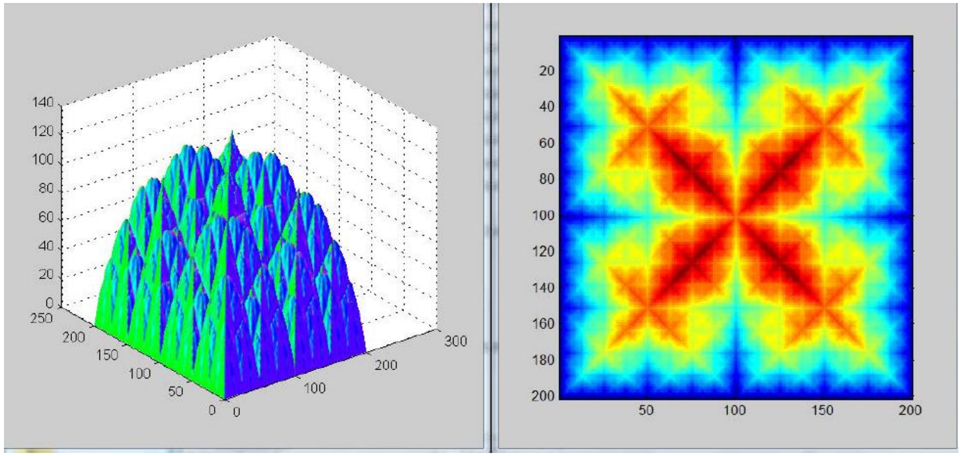
(America)

**Niagara Project**

# Matrix Art

## The First Trigger of the Niagara Project

(and the Main Theme in a Challenge Seminar in Tsuyama National College of Technology)



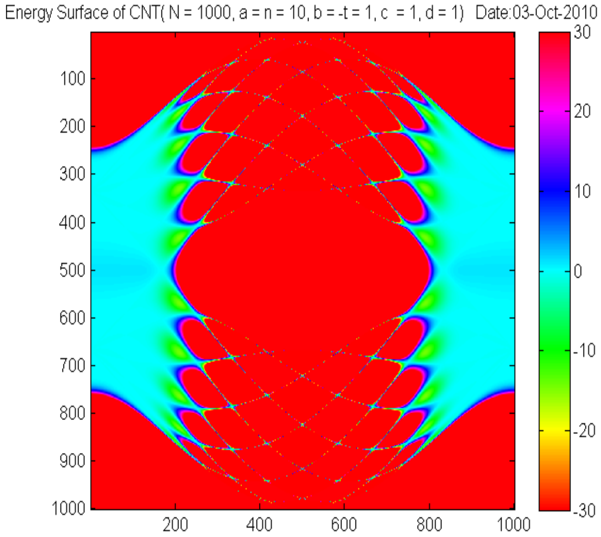



**Fig. 1**
3D Magic Mountain and 2D Magic Mountain

**Fig. 2**
Carbon Nanotube Energy Band

# Corning Glass Museum

## The Second Trigger of the Niagara Project

On the way to Niagara Water Fall, the idea of the joint project was born


Niagara

Penn College

Corning Glass Museum

# **Interdisciplinary Region**
# between Science, Technology, and Art


Corning Glass Museum → Idea of the Niagara Project

(The second trigger of the Niagara Project)

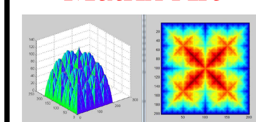
## **Niagara Project** (2010 ~) : Extension of **Fukui Project** (1992 ~)

- International (Japan, Canada, France)
- Interdisciplinary (Math & Science & Art)
- Inter-generational (Experts & Students)

Kenichi Fukui (1918 – 1998,  Nobel Chemistry Prize 1981)

**Fukui Project**
(Foundation of the Niagara Project)

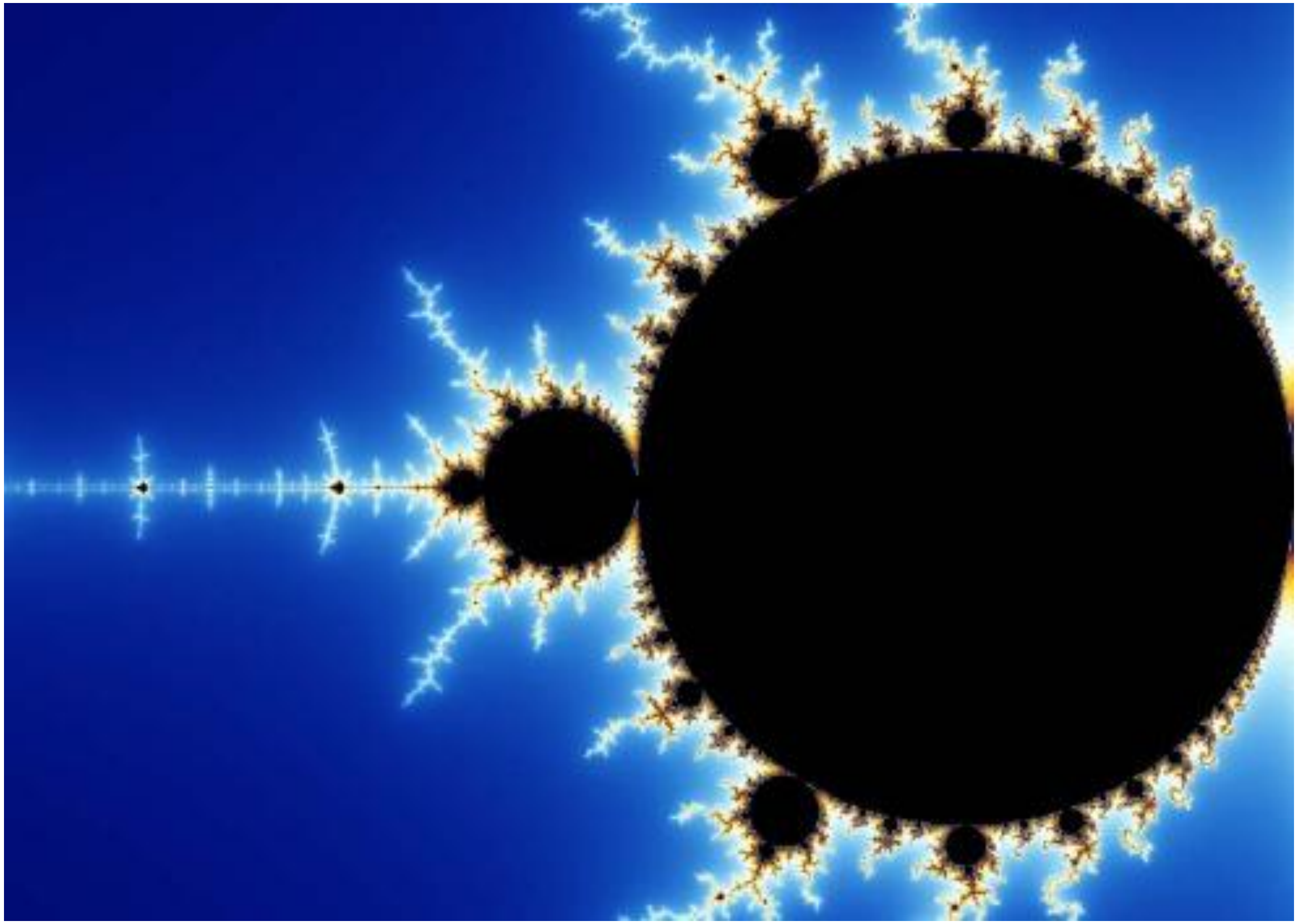
Matrix Art



Corning Art

**Niagara Project**

# **Self-Similarity** （Mandelbrot set）



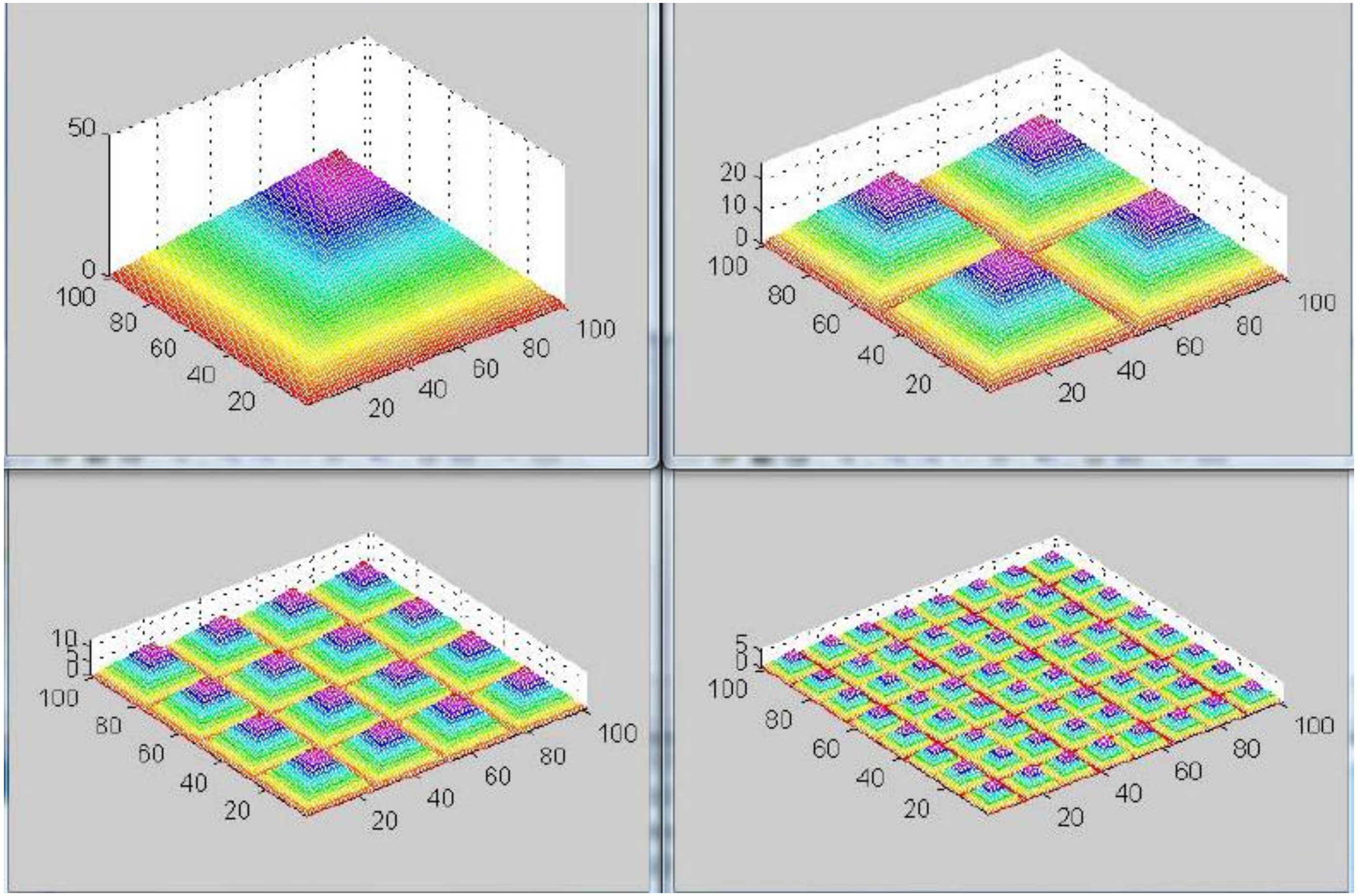
## How to make Magic Mountain

$$\text{Magic Mountain} = \sum_{n=0}^{\infty} \text{Pyramid}(n)$$

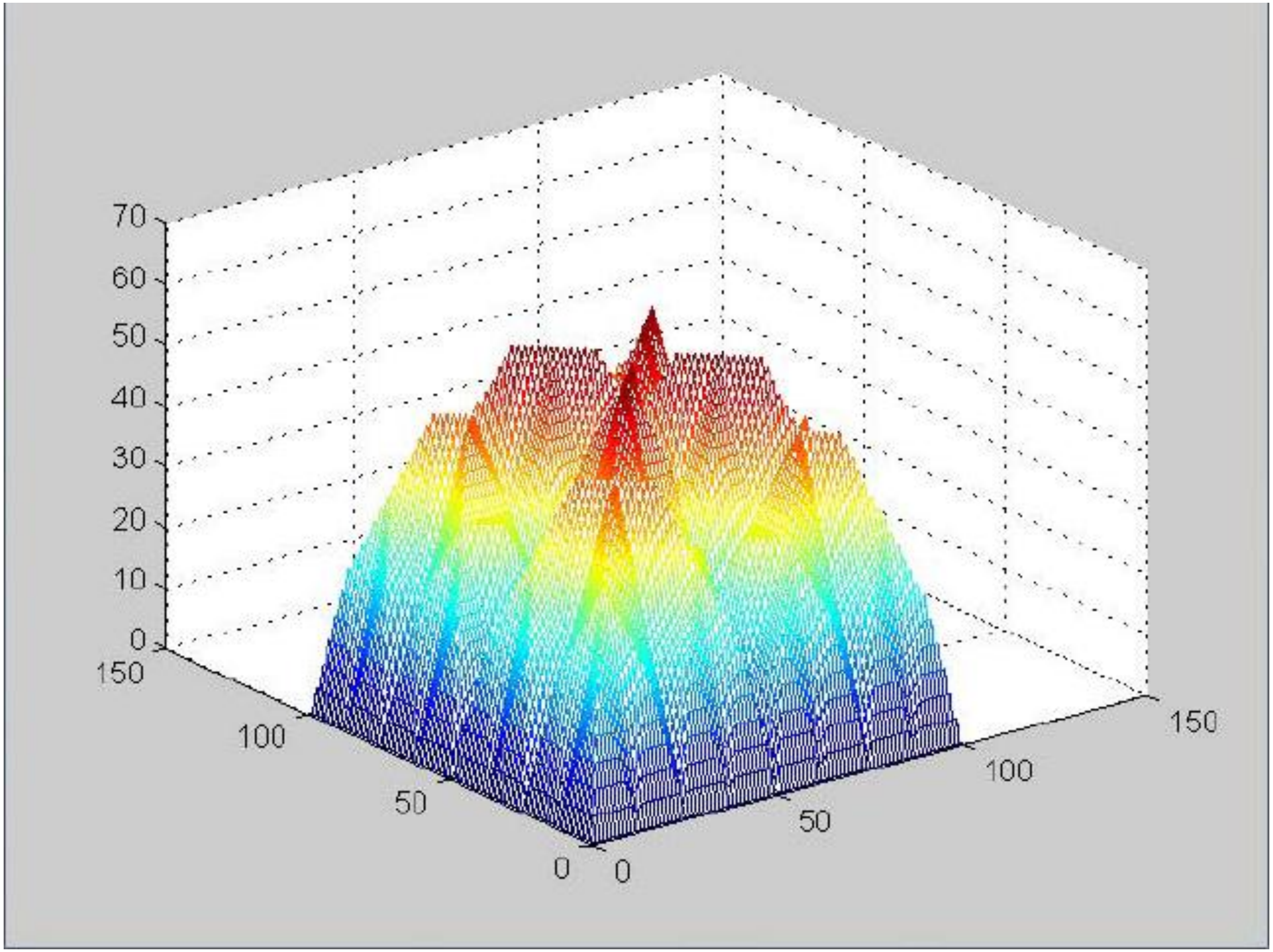
In other words, Magic Mountain is the infinite sum of the pyramid functions called Pyramid($n$) which are given in what follows.

**Pyramid function**: Pyramid($n$)

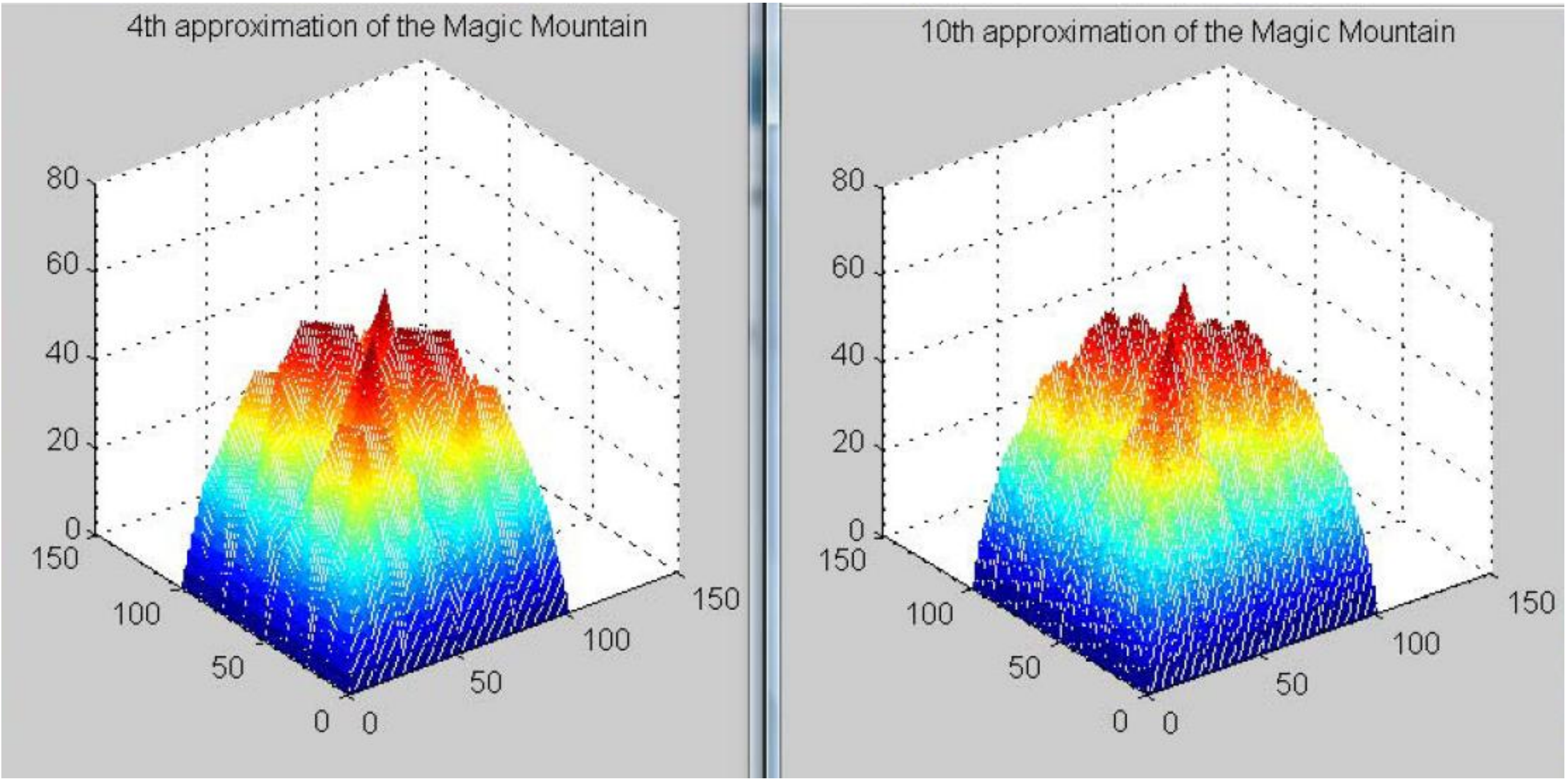
$n = 0, 1, 2, 3$



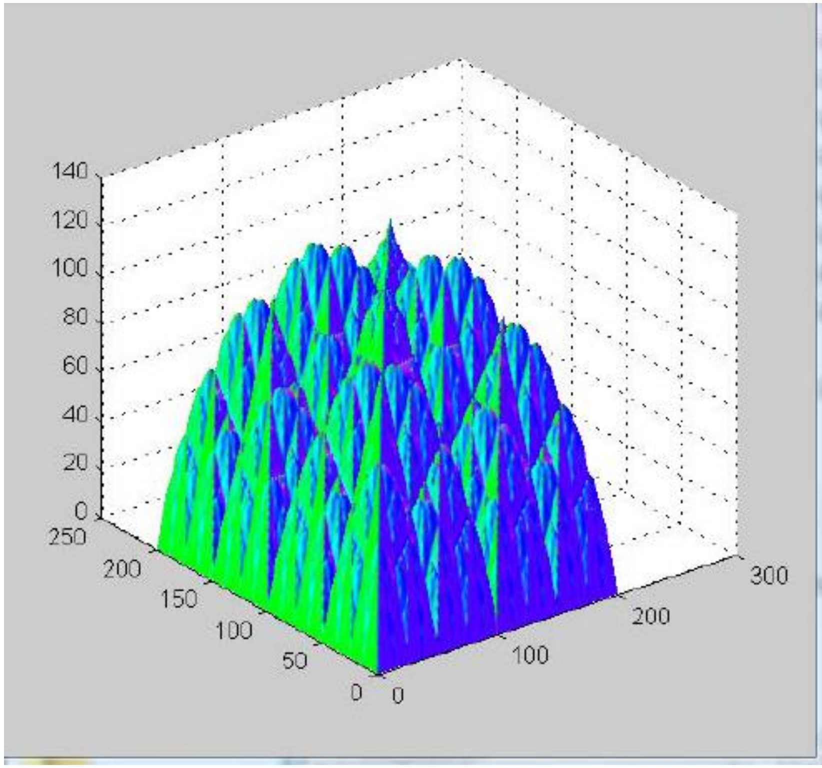
## **The 4th Approximation of Magic Mountain** =
Pyramid(0) + Pyramid(1) + Pyramid(2) + Pyramid(3)

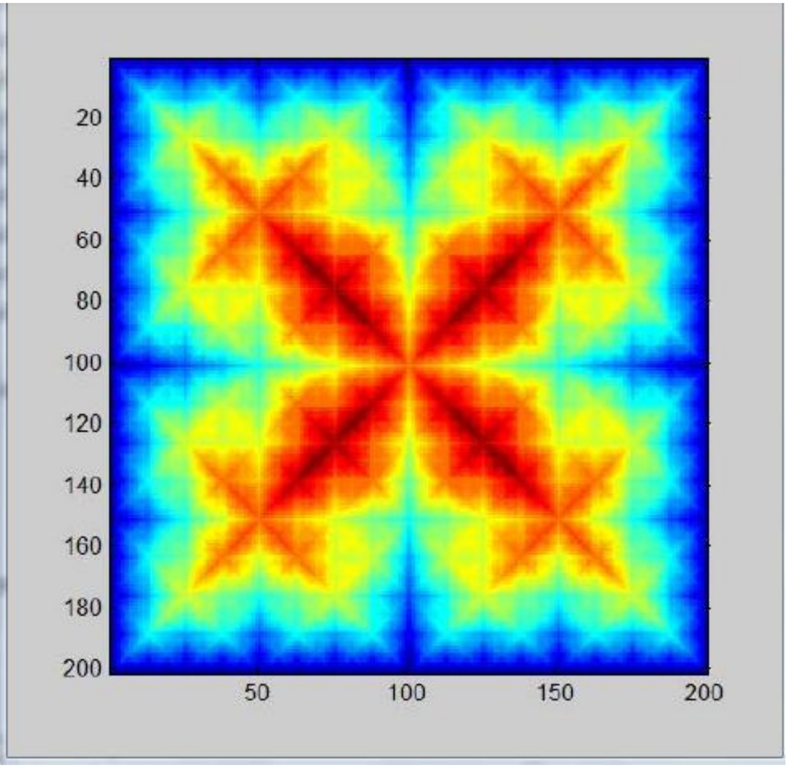


# The 4th and 10th Approximations of Magic Mountain



## The 30th Approximation of Magic Mountain and its Matrix Pattern: 2D Magic Mountain (Logo of the Niagara Project)



# Publications

The Matrix Art employed in Tsuyama NCT has recently been published in the international Journal of Mathematical Chemistry, Springer

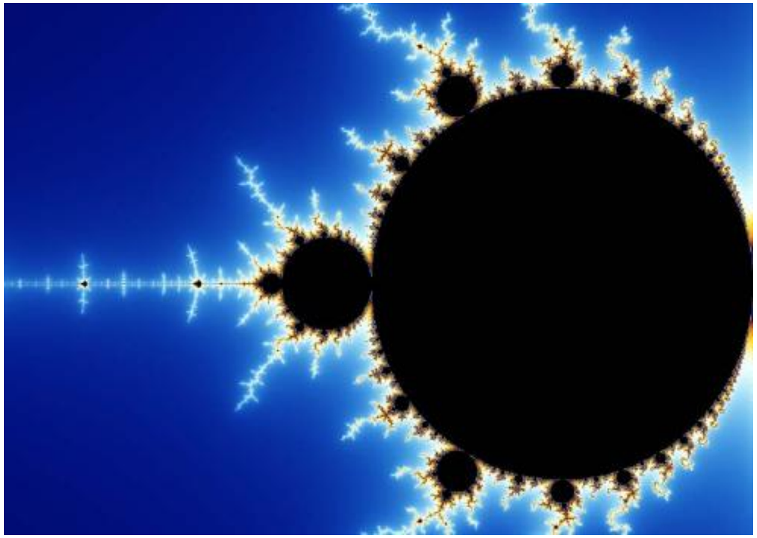
S. Arimoto, Fundamental notions for the second generation Fukui project and a prototypal problem of the normed repeat space and its super spaces, J. Math. Chem. 49 (2011) 880].

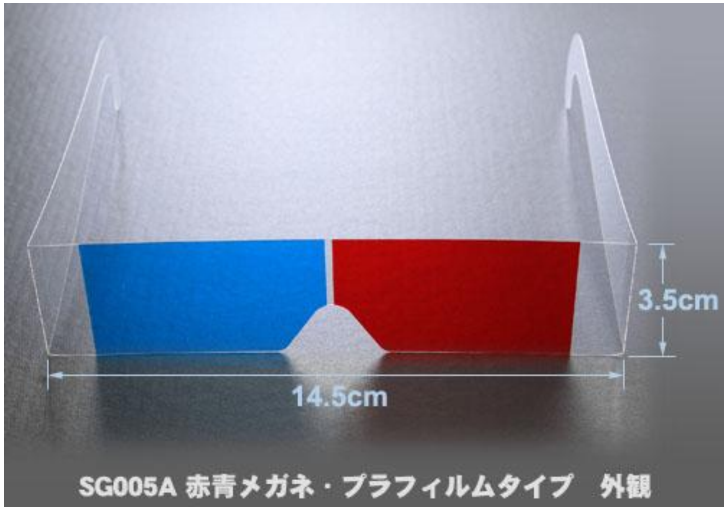
S. Arimoto, M. Spivakovsky, E. Yoshida, K.F. Taylor, and P.G. Mezey, Proof of the Fukui conjecture via resolution of singularities and related methods. V, J. Math. Chem. (2010) Digital Object Identifier (DOI) **10.1007/s10910-011-9852-1**

**Springer Online**

# Movie of Matrix Art

# Self-Similarity　自己相似性
（Mandelbrot set　２Dフラクタル）



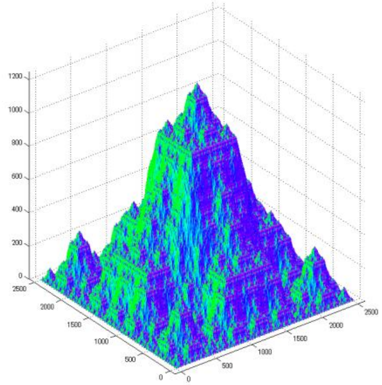



**問題１**：魔の山を「進化」させて、優美な3Dフラクタルはできないか？

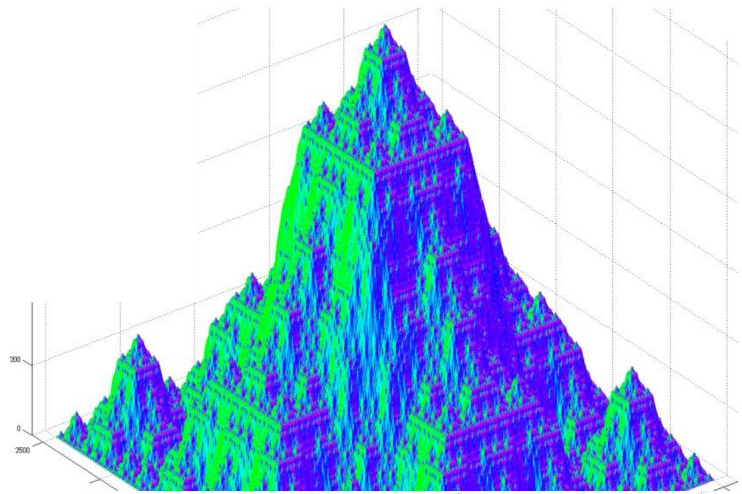
**問題２**：赤青メガネで立体視できる3Dフラクタルはできないか？

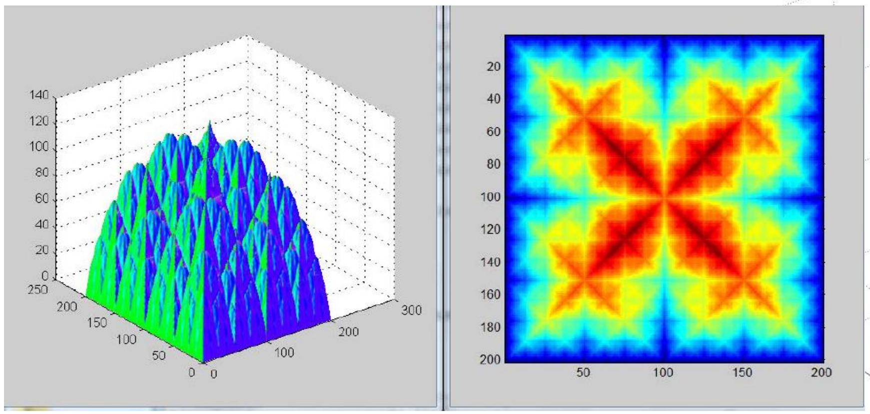
# MagicMt( $\pi$ )

問題１の答

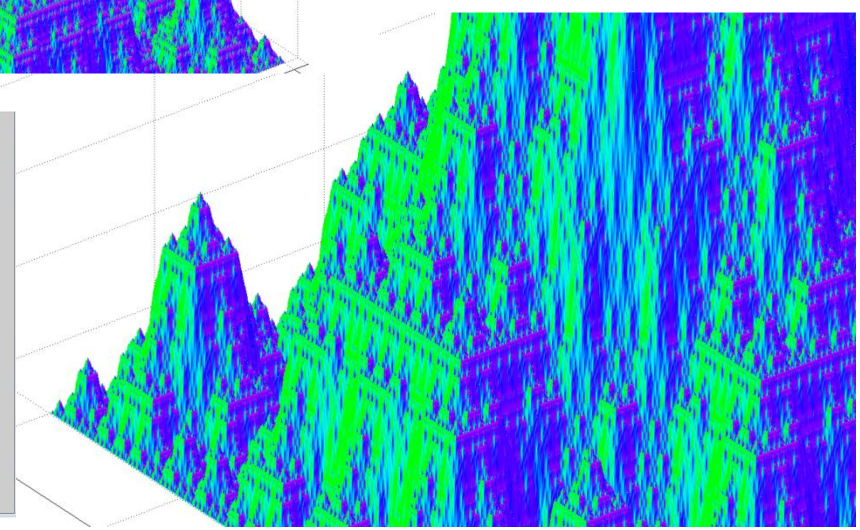


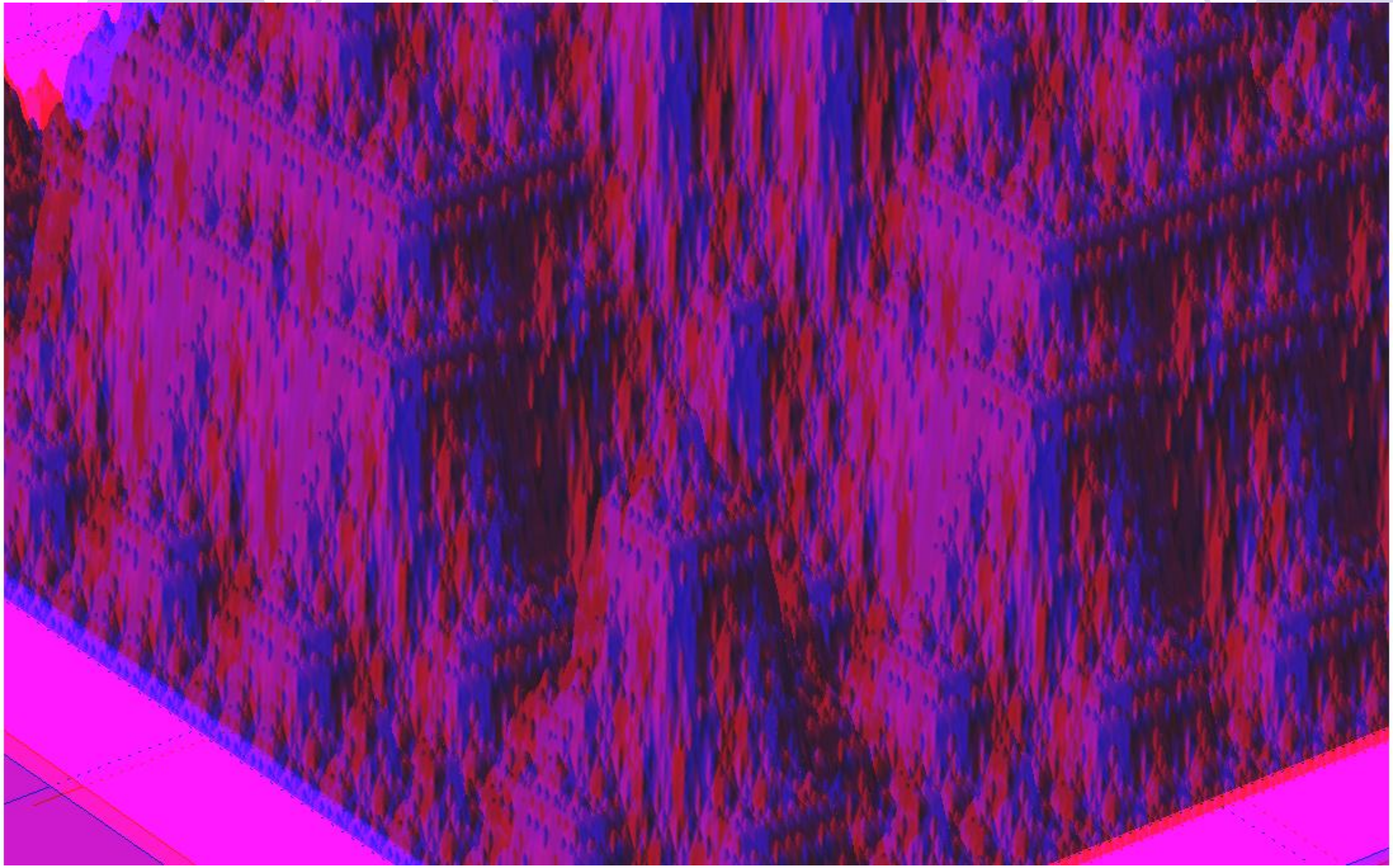
進化






**MagicMt(0)**



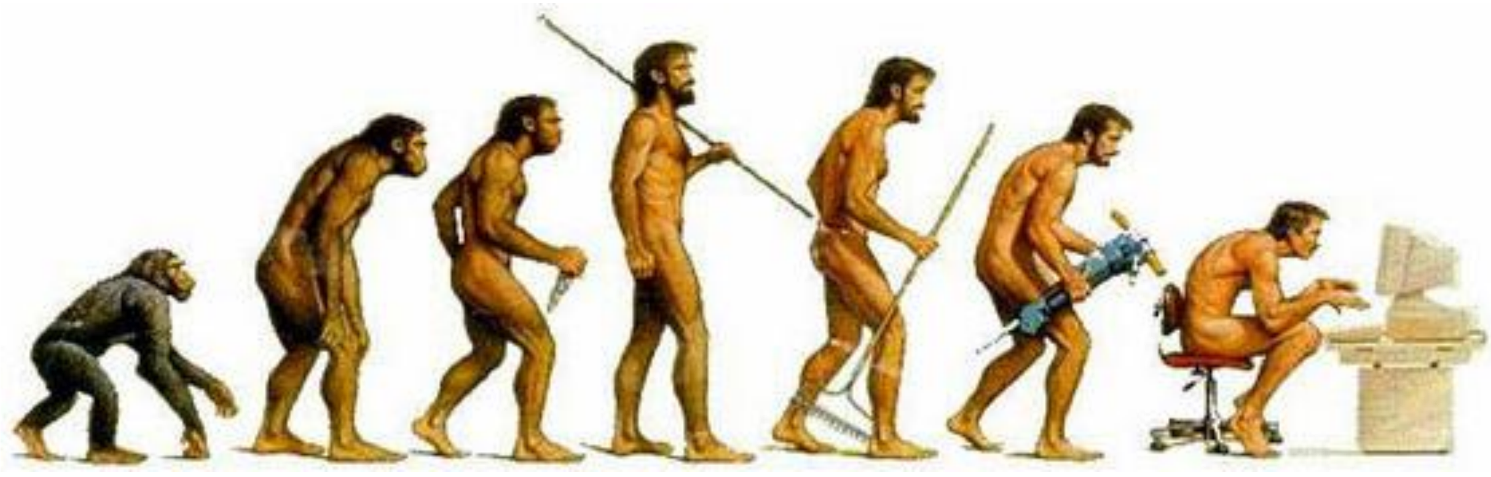


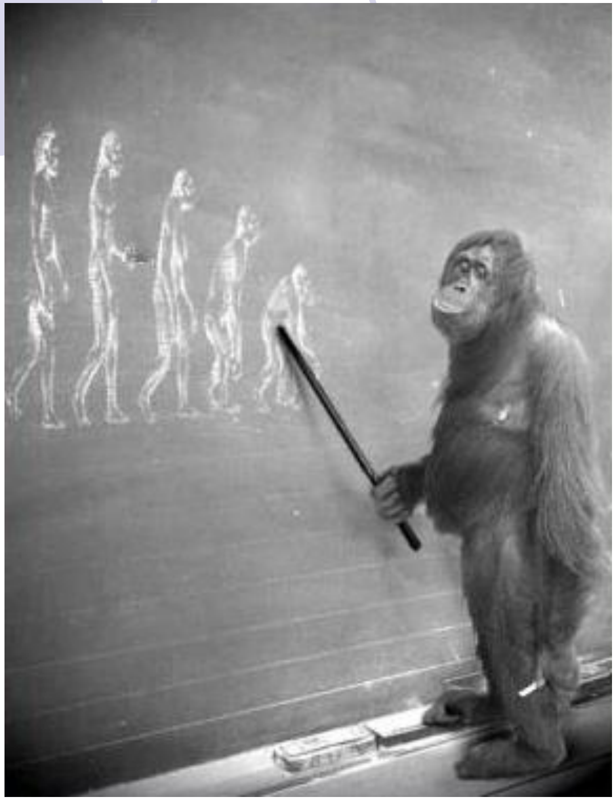
**Anaglyph of  MagicMt($\pi$)  =３Ｄ・フラクタル城**
（問題２の答）

# Time Evolution to $\text{MagicMt}(\pi)$

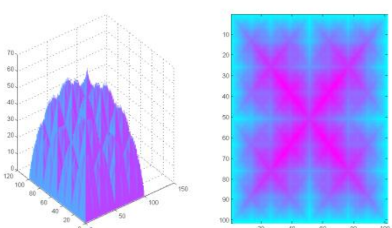
$$\text{MagicMt}(\theta) = \sum_{n=0}^{\infty} \cos(n\theta)\,\text{Pyramid}(n)$$

$$\theta = \frac{0\pi}{30}, \frac{1\pi}{30}, \frac{2\pi}{30}, \frac{3\pi}{30}, \frac{4\pi}{30}, \frac{5\pi}{30}, \ldots, \frac{29\pi}{30}, \frac{30\pi}{30}$$

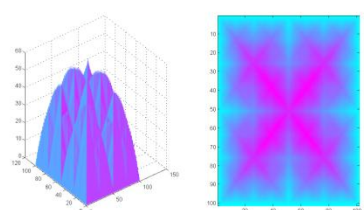




# Time Evolution for the first 9 minutes



０　分



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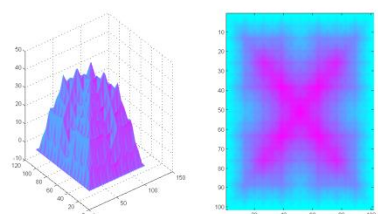
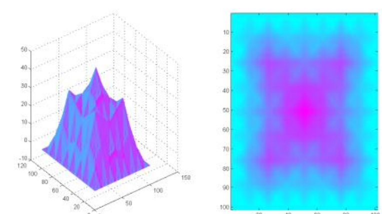
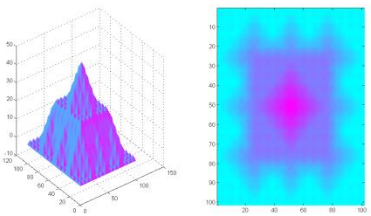


６　分

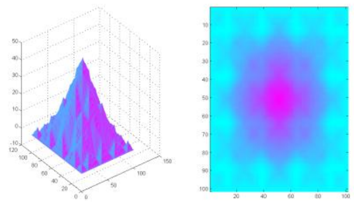


９　分

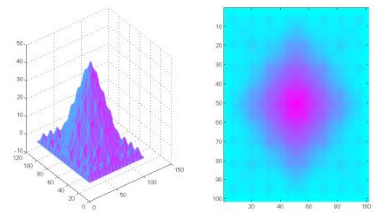
# Time Evolution to the Castle



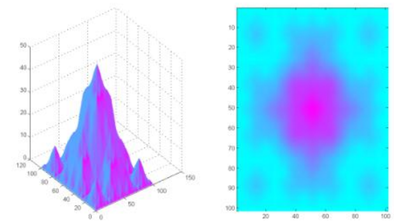
１２ 分



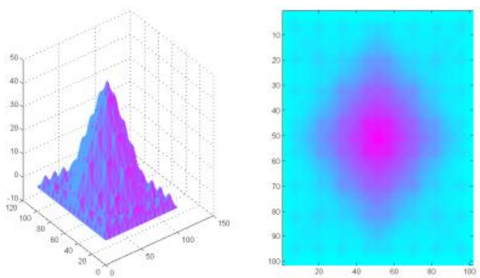
１５ 分



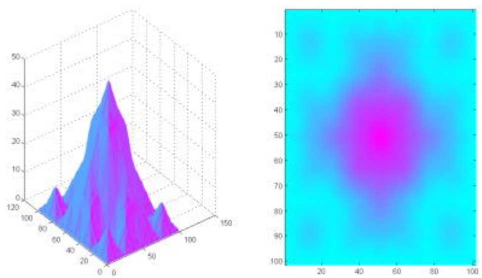
１８ 分



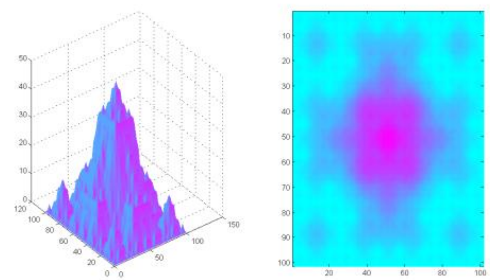
２１ 分



２４ 分



２７ 分



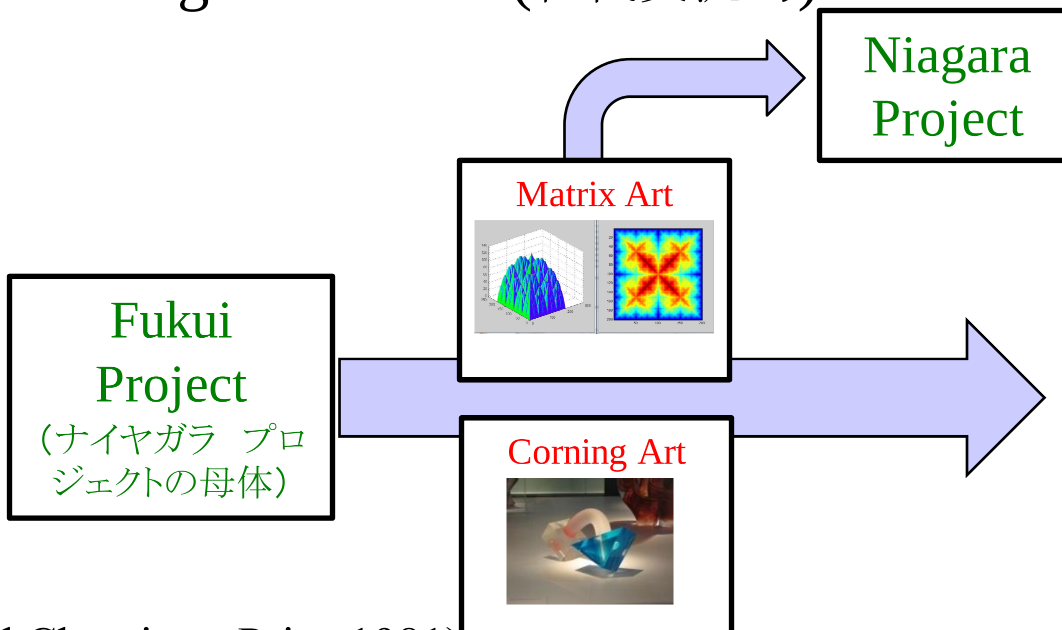
３０ 分

$$\mathrm{MagicMt}\left(\frac{12\pi}{30}\right) \Rightarrow \mathrm{MagicMt}\left(\frac{30\pi}{30}\right)$$

# Fukui Project (1992 ~) : Niagara Project (2010 ~) の母体

A project with：
Central theory： Repeat Space Theory （RST)
Guiding conjecture： Fukui Conjecture

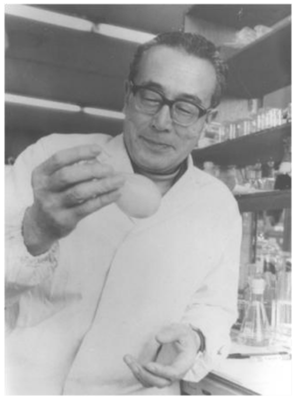
- International (Japan, Canada, France)
- Interdisciplinary (Math & Science & Art)
- Inter-generational (世代交流的)



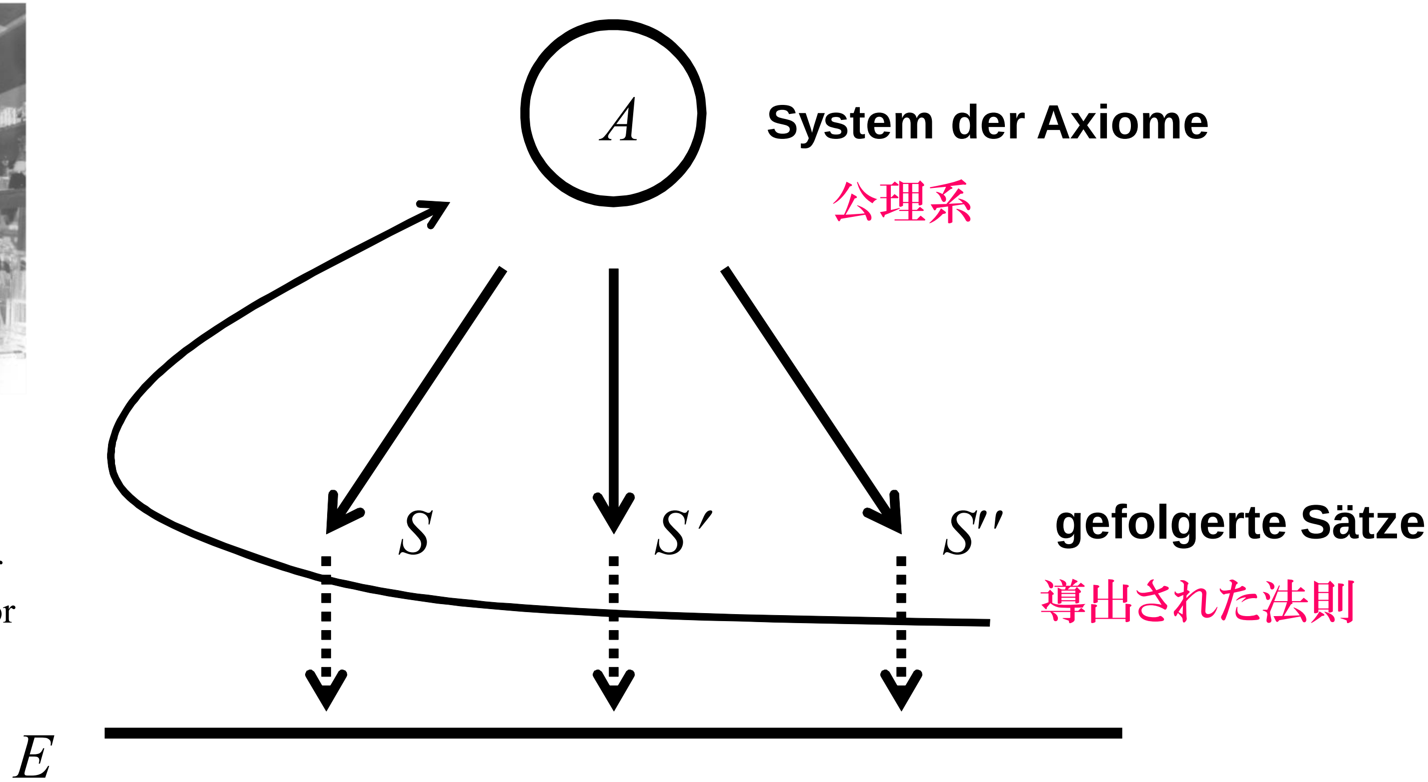

Kenichi Fukui (1918 – 1998, Nobel Chemistry Prize 1981)

# Repeat Space Theory（RST)

# 理論と経験の巡回的相互作用
(From: A. Einstein’s Hand-writing)



Haruo Shingu
(1907-1988)
Fukui’s teacher
and collaborator




**Mannigfaltigkeit der unmittelbaren (Sinnen-) Erlebnisse**

直接（感性）経験の多様性

## Mathematical Fields Related to the RST

The RST uses techniques from the following branches of mathematics:

- **Functional Analysis** (especially the theory of **Banach spaces, Operator Algebra**)
- **General Topology**
- **Ring Theory** (especially the theory of **UFDs**)
- **Algebraic Geometry** (especially the theory of **Resolution of Singularities** )

Logical Interface Language (LI L)

## Fundamental Molecular Properties Investigated by Using the Repeat Space Theory (RST)

- **Zero-point Vibrational Energy** (ZPVE) : $E_{zero}$ (**Prototypical**)
- **Internal Energy**: $E_T$
- **Heat Capacity**: $C_v$
- **Total Pi Electron Energy** (TPEE): $E_\pi$
- **Superdelocalizability**: $S_r$ (**notion from the Frontier Electron Theory** )

# Logical Interface

**Definitions 5.1.** Let $\boldsymbol{L}$ denote the topological space with the underlying set $\{\boldsymbol{T}, \boldsymbol{F}\}$ and the system of open sets $o_T = \{\varnothing, \{\boldsymbol{F}\}, \{\boldsymbol{T}, \boldsymbol{F}\}\}$. The topological space $\boldsymbol{L}$ is called the *logical space*. Let $X, X_1, \ldots, X_n$ be topological spaces, let $\pi : X \to \boldsymbol{L}$ be a continuous mapping, let $\rho_1 : X \to X_1, \ldots, \rho_n : X_{n-1} \to X_n$, and $\rho_{n+1} : X_n \to \boldsymbol{L}$ be continuous mappings such that the following diagram

$$\begin{tikzcd} X \arrow[rrrr, "\pi"] \arrow[d, "\rho_1"'] & & & & \boldsymbol{L} \\ X_1 \arrow[r, "\rho_2"'] & X_2 \arrow[r, "\rho_3"'] & \cdots \arrow[r] & X_{n-1} \arrow[r, "\rho_n"'] & X_n \arrow[u, "\rho_{n+1}"'] \end{tikzcd}$$

is commutative, i.e., such that

$$\pi = \rho_{n+1} \circ \cdots \circ \rho_1. \tag{5.1}$$

The mapping $\pi$ is called a *logical interface* on $X$. Each $\rho_i$, $1 \leqslant i \leqslant n+1$, is called a *component* of $\pi$. Equality (5.1) is called a *component analysis* of $\pi$.

**Notation 3.1.** Let $I = [a, b](a, b \in \mathbb{R}, a < b)$ denote a closed interval.

$V_I(\varphi)$: the total variation of a real-valued function $\varphi$ on $I$, i.e.,

$$V_I(\varphi) = \sup_{\Delta} \sum_{i=1}^{n} \left|\varphi(t_i) - \varphi(t_{i-1})\right|, \tag{3.11}$$

$$\Delta\colon\ a = t_0 \leqslant t_1 \leqslant \cdots \leqslant t_n = b.$$

$BV(I)$: the set of all real-valued functions of bounded variation on $I$, i.e., the set of all real-valued functions $\varphi$ on $I$ such that $V_I(\varphi) < \infty$.

$CBV(I)$: the normed space of all real-valued continuous functions of bounded variation on $I$ equipped with the norm given by

$$\|\varphi\| = \sup\left\{\left|\varphi(t)\right|\colon\ t \in I\right\} + V_I(\varphi). \tag{3.12}$$

$AC(I)$: the normed space of all real-valued absolutely continuous functions on $I$ equipped with the norm given by

$$\|\varphi\| = \sup\left\{\left|\varphi(t)\right|\colon\ t \in I\right\} + V_I(\varphi). \tag{3.13}$$

$P(I)$: the set of all polynomial functions with real coefficients defined on $I$.

$\overline{\phantom{x}}$: the closure operation on a topological space.

$\boldsymbol{B}(X, Y)$: the normed space of all bounded linear operators from a normed space $X$ to a normed space $Y$.

$CBV(I)^*$: the dual space of $CBV(I)$, i.e.,

$$CBV(I)^* = \boldsymbol{B}\big(CBV(I), \mathbb{R}\big). \tag{3.14}$$

$AC(I)^*$: the dual space of $AC(I)$, i.e.,

$$AC(I)^* = \boldsymbol{B}\big(AC(I), \mathbb{R}\big). \tag{3.15}$$

*Proof.* (i) Under the assumptions of the theorem, consider the mapping $\pi_0 : X \to \boldsymbol{L}$ defined by

$$\pi_0(\varphi) = \begin{cases} \boldsymbol{T} & \text{if } \{\tau_N(\varphi)\} \text{ is a Cauchy sequence,} \\ \boldsymbol{F} & \text{if } \{\tau_N(\varphi)\} \text{ is not a Cauchy sequence.} \end{cases} \tag{4.11}$$

Then, because $\mathscr{B}$ is complete, we see that

$$\pi = \pi_0. \tag{4.12}$$

But, theorem 4.4 below implies that $\pi_0$ is continuous. Hence, $\pi$ is continuous.

(ii) Suppose that $X_0$ is a subset of $X$ with $\pi(X_0) = \{\boldsymbol{T}\}$. Then by (i), we have $\pi(\overline{X_0}) \subset \overline{\pi(X_0)}$. This implies that $\pi(\overline{X_0}) \subset \{\boldsymbol{T}\}$. The opposite inclusion $\pi(\overline{X_0}) \supset \{\boldsymbol{T}\}$ is obvious.

(iii) By (ii), it remains to prove that the operator $\tau$ is linear and bounded. Since $\tau_N$ is linear, the linearity of $\tau$ is obvious. The boundedness follows from the relations:

$$\begin{aligned} \|\tau(\varphi)\| &= \left\| \lim_{N\to\infty} \tau_N(\varphi) \right\| \\ &= \lim_{N\to\infty} \|\tau_N(\varphi)\| \\ &= \underline{\lim}_{N\to\infty} \|\tau_N(\varphi)\| \\ &\leqslant \left( \underline{\lim}_{N\to\infty} \|\tau_N\| \right) \|\varphi\| \\ &\leqslant \left(\sup\{\|\tau_N\| : N \geqslant 1\}\right) \|\varphi\|. \end{aligned} \tag{4.13}$$

$\square$

# Repeat Space Theory (RST) and the Developmental Theory of Cultural Anthropology

Structure 1

Structure 2

Structure 3

Structure 4

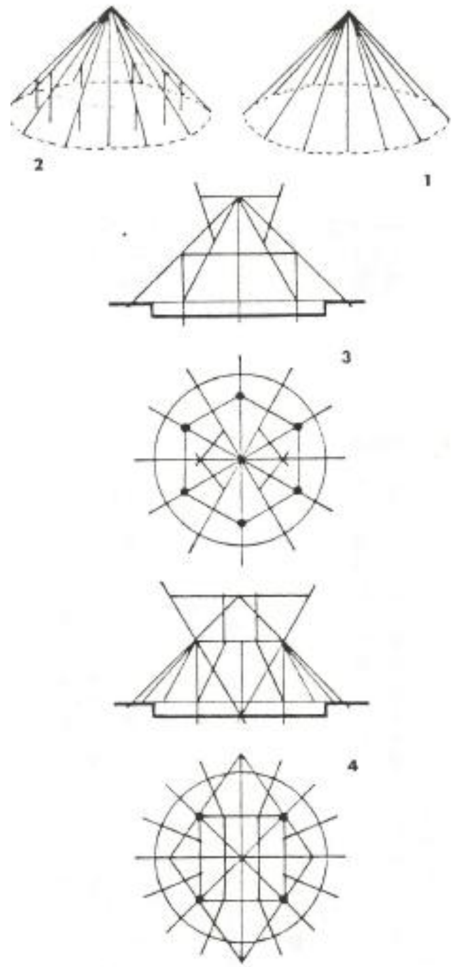



図1 原始住居構造の発達過程の復元案（藤島 1951）

# Repeat Spaces

RS1. Original Repeat Space $X_r(q)$: Jordan algebra

RS2. Extended Repeat Space $Y_r(q)$: algebra

RS3. Generalized Repeat Space $X_r(q, d)$: algebra

RS4. Normed Repeat Space $Xr(q, d, p)$* : Banach algebra, C*-algebra

* Published: Normed repeat space and its superspaces: fundamental notions for the second generation Fukui project, S. Arimoto, J. Math Chem **46**, 586 (2009).

Published: Proof of the Fukui conjecture via resolution of singularities and related methods I-V, S. Arimoto, etal, J. Math Chem **47**, 856 (2010).

# Magic Mountain Castle

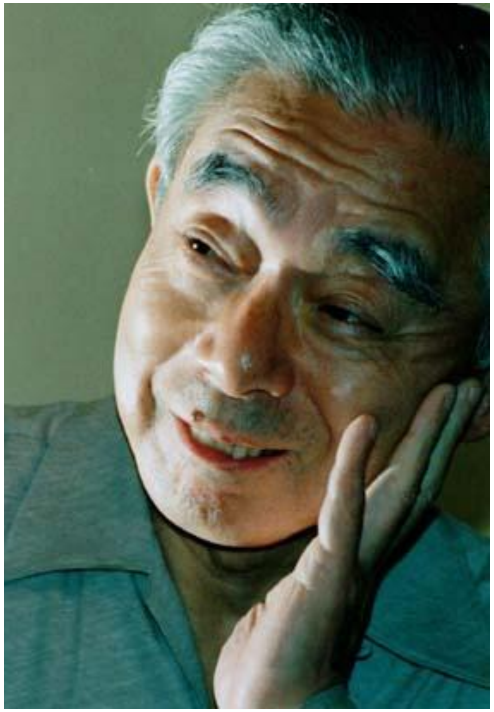
3D Object having
Self Similarity

Thank you!

御清聴有難うございました。

# Outside the space $AC(I)$

ALTEC: The ALT cannot be extended to $C(I)$.



Kenichi Fukui (1918 - 1998)

**The Fukui Conjecture (Main Part).**

Let $\{M_N\}$ be a fixed element of $X_r(q)$ (the repeat space with block-size $q$), and let $I$ be a fixed closed interval on the real line such that $I$ contains all the eigenvalues of $M_N$ for all positive integers $N$. Let $f_0$: $I \to R$ denote the function defined by

$$f_0(t) = \frac{\hbar}{2}|t|^{1/2}.$$

Then, there exist real numbers $\alpha$ and $\beta$ such that

$$f_0(\lambda_i(M_N)) = \mathrm{Tr}\, f_0(M_N) = \alpha N + \beta + o(1)$$

as $N \to \infty$.

## The Functional Asymptotic Linearity Theorem that proves the Fukui conjecture

**Theorem A** (Functional ALT, $X_r(q)$-version). Let $\{M_N\} \in X_r(q)$ be a fixed repeat sequence, let $I$ be a fixed closed interval compatible with $\{M_N\}$. Then, there exist functionals $\alpha, \beta \in AC(I)^* = \boldsymbol{B}(AC(I), R)$ such that

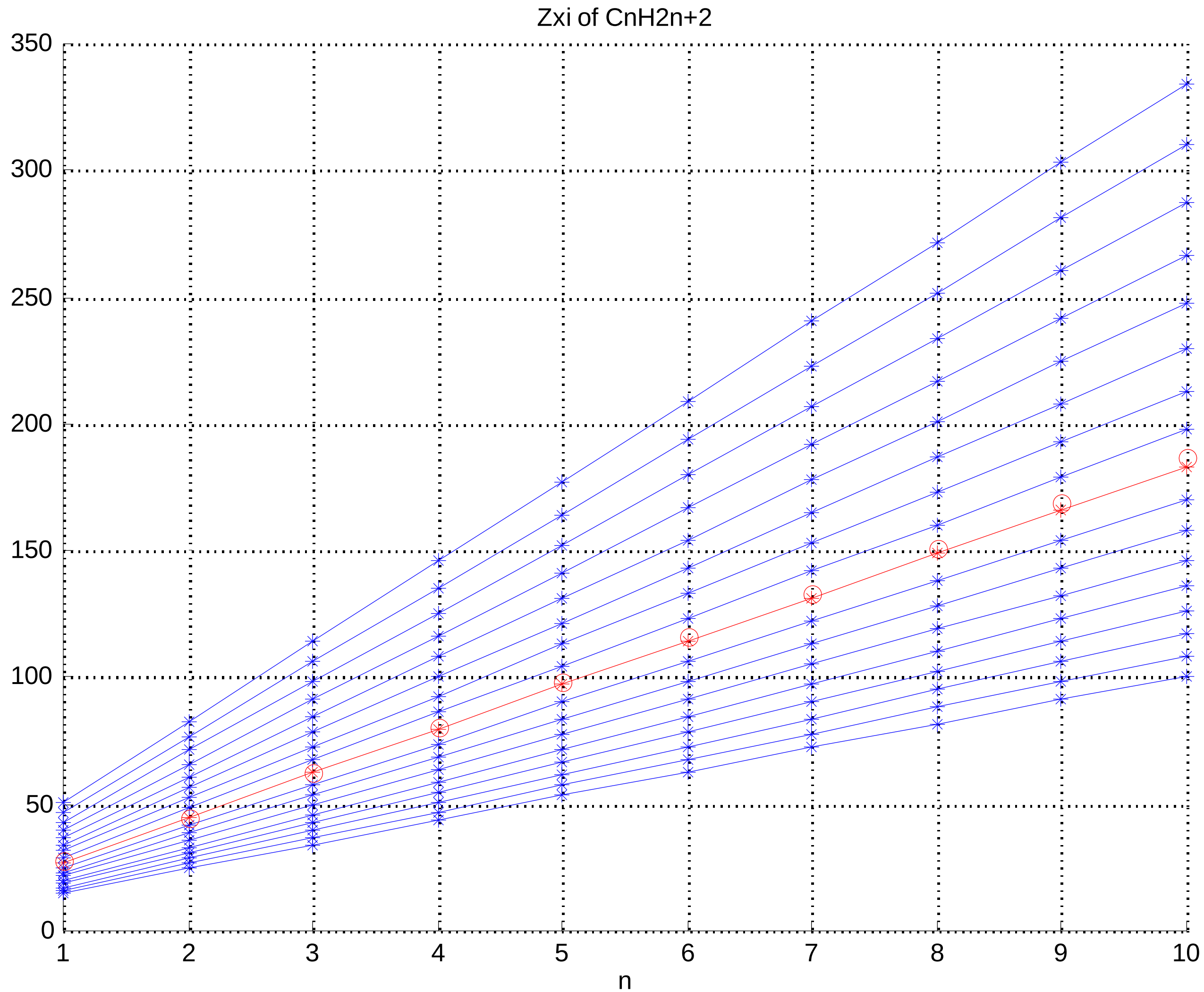
$$\mathrm{Tr}\,\varphi(M_N) = \alpha(\varphi)N + \beta(\varphi) + o(1)$$

as $N \to \infty$, for all $\varphi \in AC(I)$.

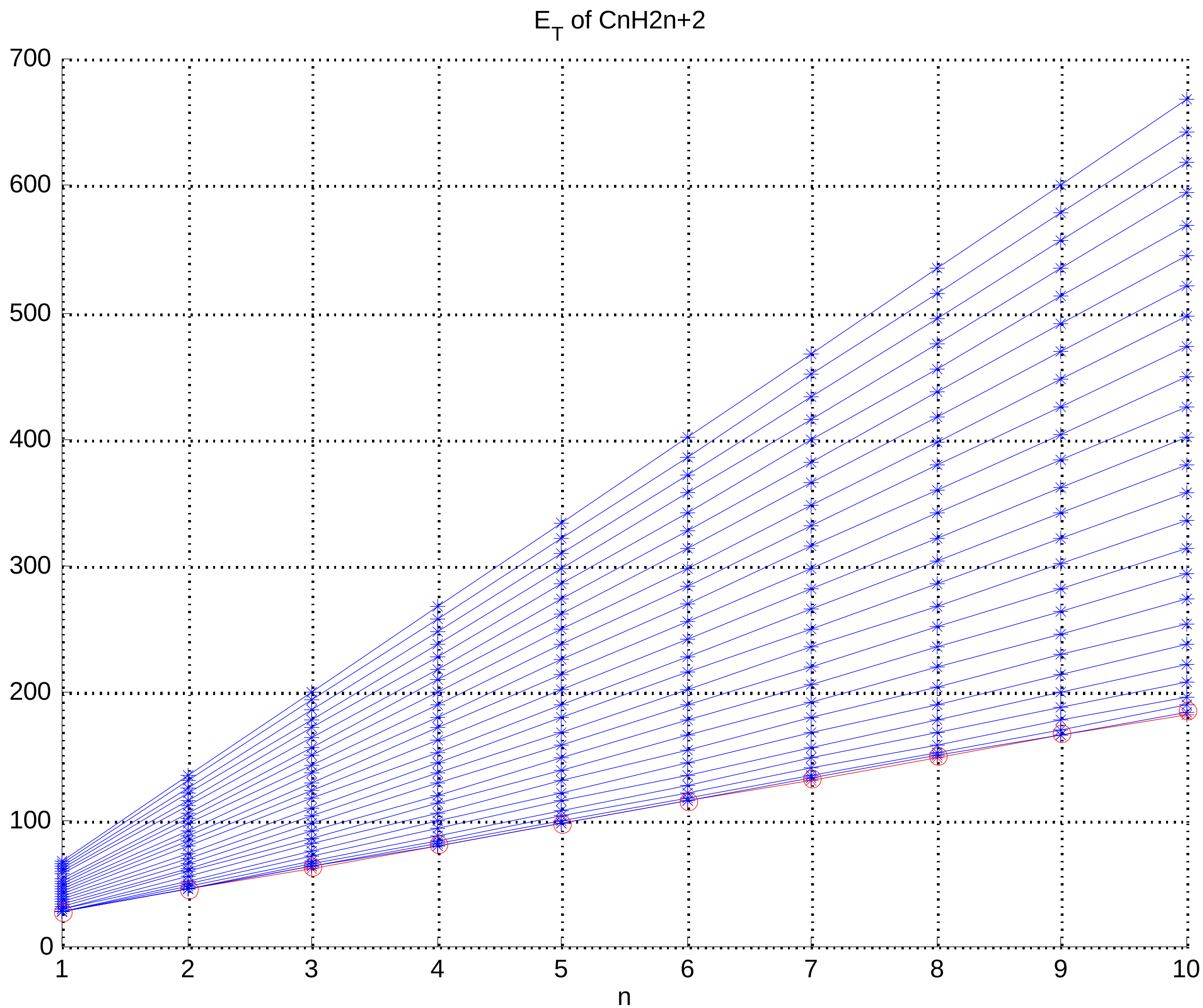
Local Analyticity Proposition => Functional ALT => Fukui Conjecture

特異点解消

# $Z_\xi$ Plotting of $C_n H_{2n+2}$ $(0.46 \le \xi \le 0.54)$

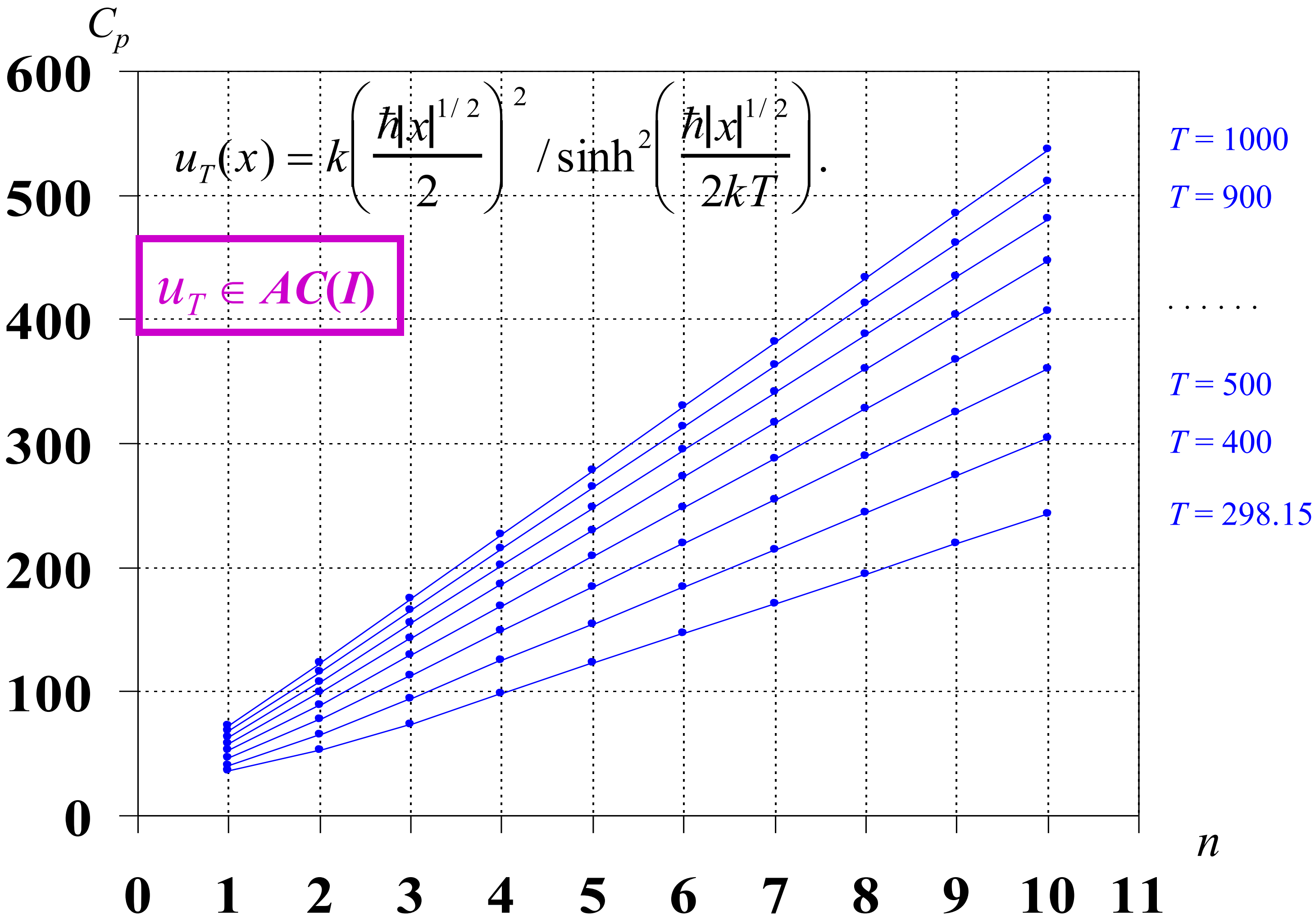


# Internal Energy of $C_n H_{2n+2}$ as $T \to 0$ (°K)

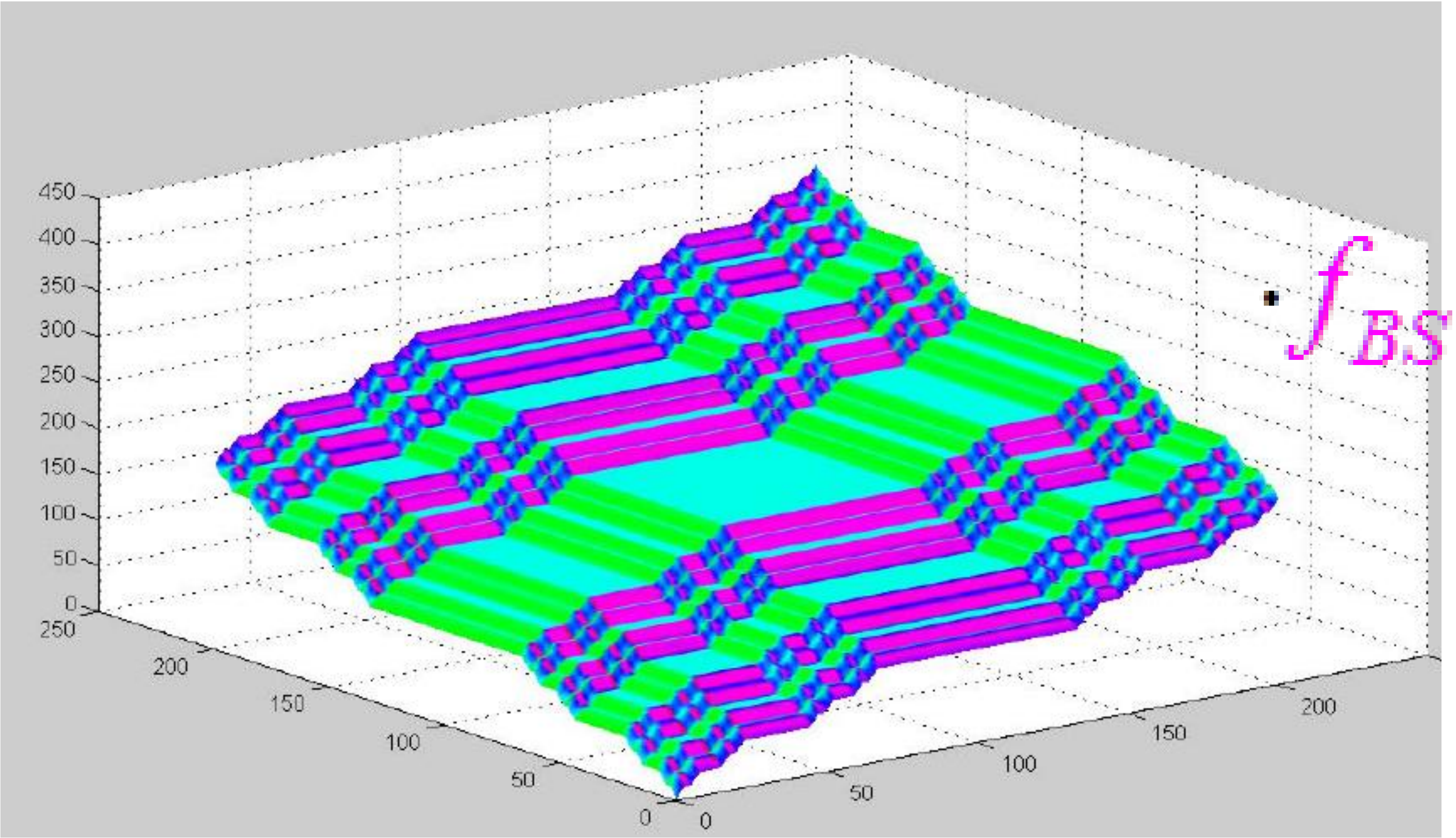


# Molar Heat Capacity of $C_n H_{2n+2}$

# $T$ = 298.15, 400, 500, …, 1000 (°K).

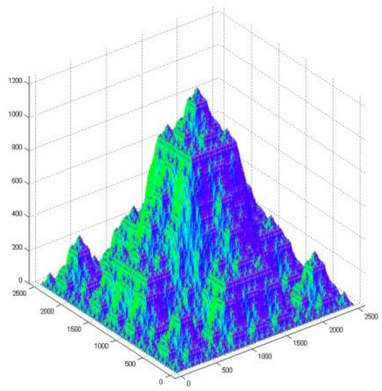


## Buddha's Terrace (from F-project) to investigate the ALTEC (from Challenge Seminar with Mr. T. Fukuda)

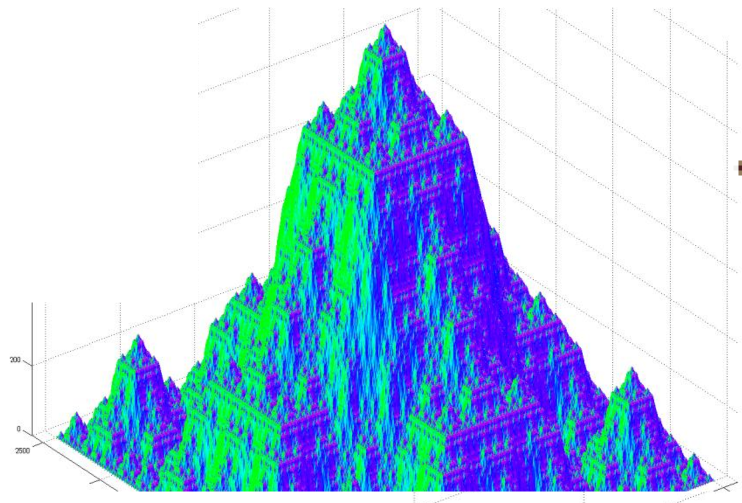



ALTEC: The ALT cannot be extended to $C(I)$. Proved!

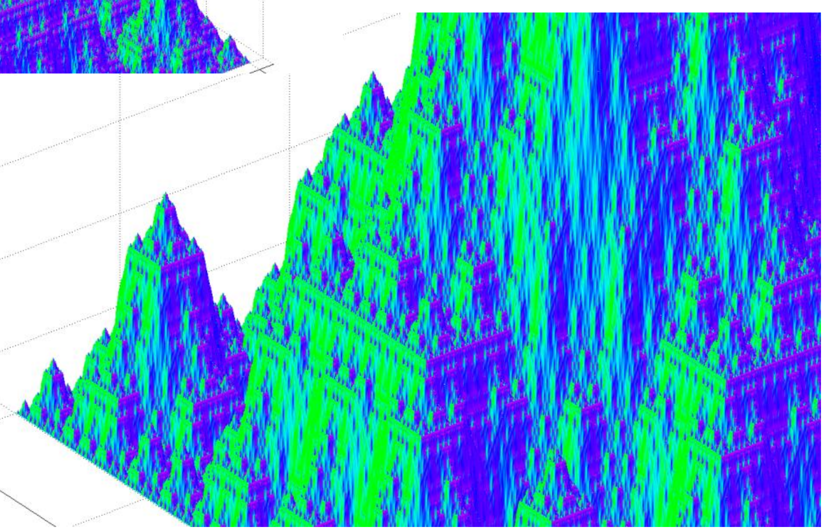
# MagicMt( $\pi$ )

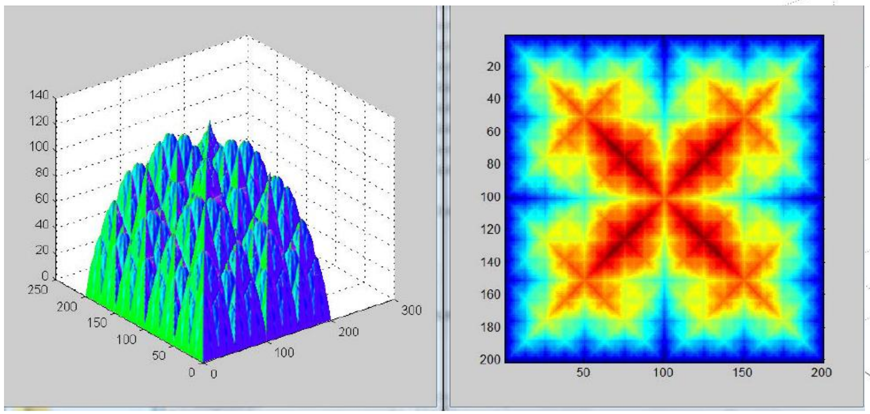


進化



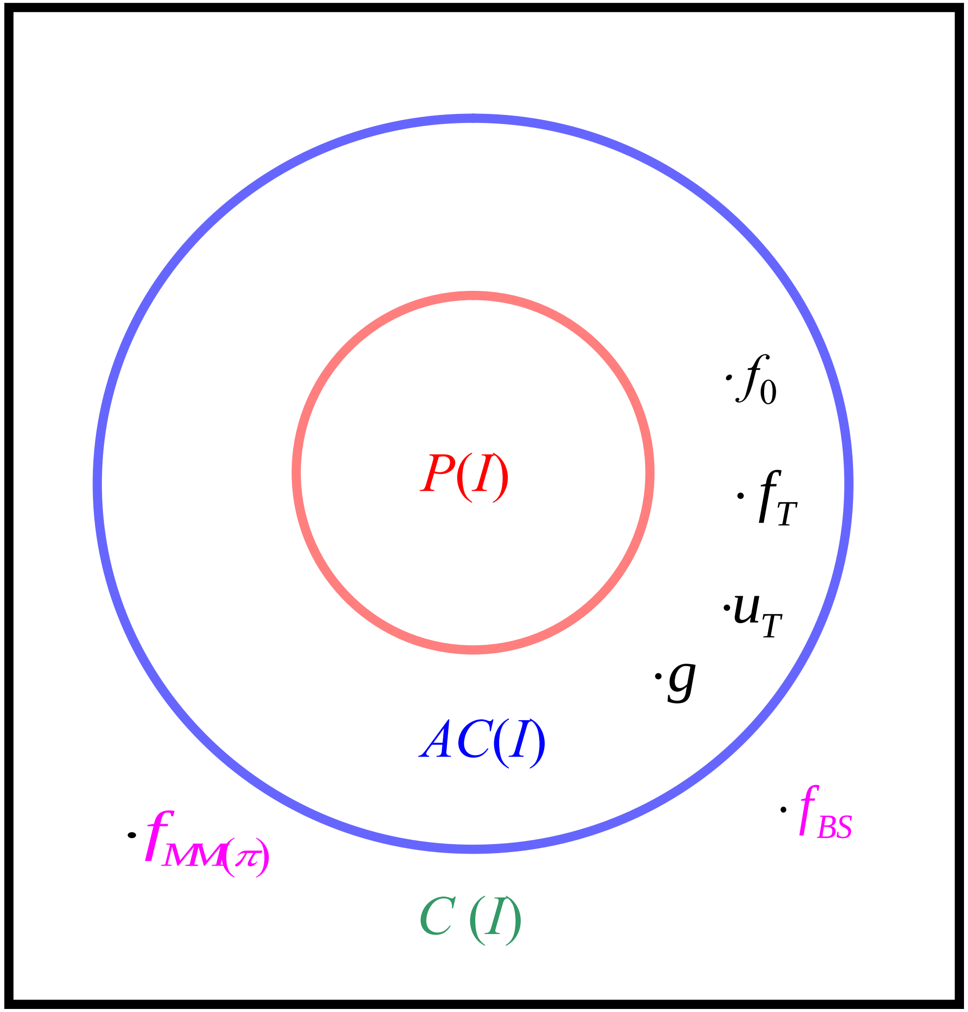
- $f_{MM(\pi)}$






**MagicMt(0)**

# Banach Space $AC(I)$



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## Niagara Project