ALT Extension Conjecture (ALTEC) in Niagara & Fukui Project

－Science-Art Multi-angle Network
の形成に向けて

津山高専　　一般・数学　　有本　茂

Corning Glass Museum

The Origin of the Niagara Project

On the way to Niagara Water Fall, the idea of the joint project was born

Niagara

Penn College

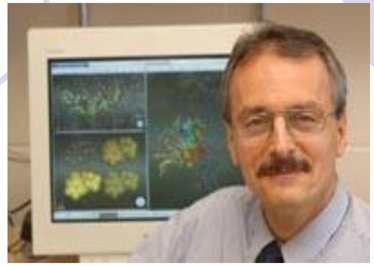
Corning Glass Museum

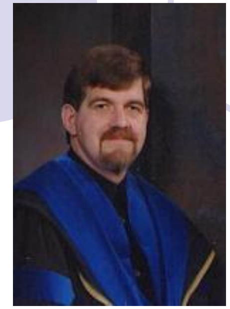
Interdisciplinary Region
between Science, Technology, and Art

Corning Glass Museum → Idea of the Niagara Project

(The second trigger of the Niagara Project)

M. Spivakovsky
(France)





P.G Mezey & Keith Taylor
(Canada)

J. Leblanc
(America)

Niagara Project in Fukui Project

New Target

T. Yamabe
(Japan)

Niagara Project (2010 ~) : Extension of **Fukui Project** (1992 ~)

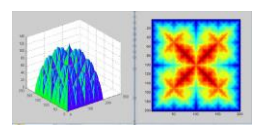
Science-Art Multi-angle Network: Further extension !

- International (Japan, Canada, France)
- Interdisciplinary (Math & Science & Art (3 kinds) & Philosophy)
- Inter-generational (Experts & Students)

Kenichi Fukui (1918 – 1998, Nobel Chemistry Prize 1981)

Fukui Project (Foundation of the Niagara Project)

Matrix Art



Corning Art

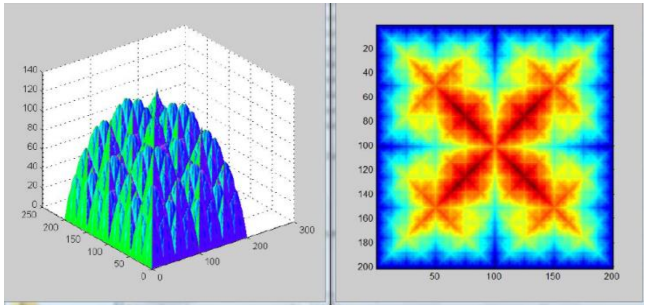
Niagara Project

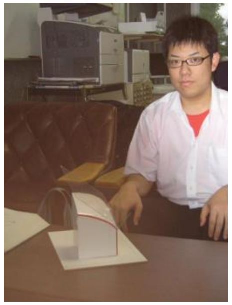
Science-Art Multi-angle Network

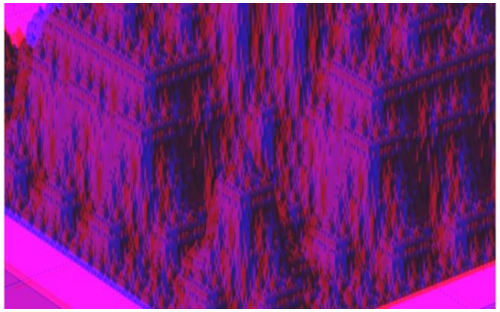
ALT Extension Conjecture

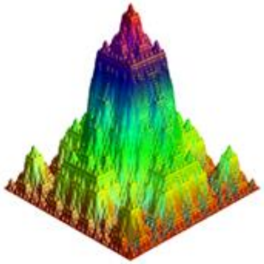
ALTEC

Matrix Art Magic Mountain （魔の山） と Tsuyama-castle（津山城関数） 津山高専 （MATLAB使用）

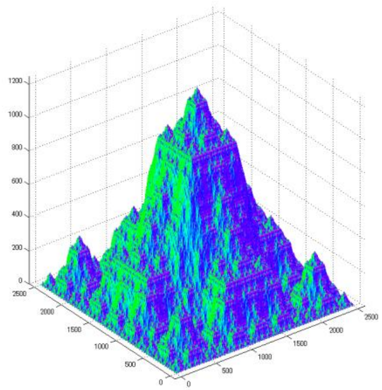




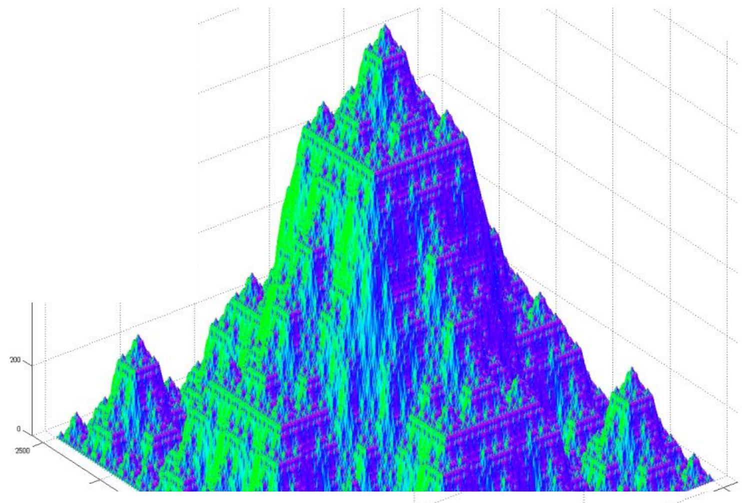


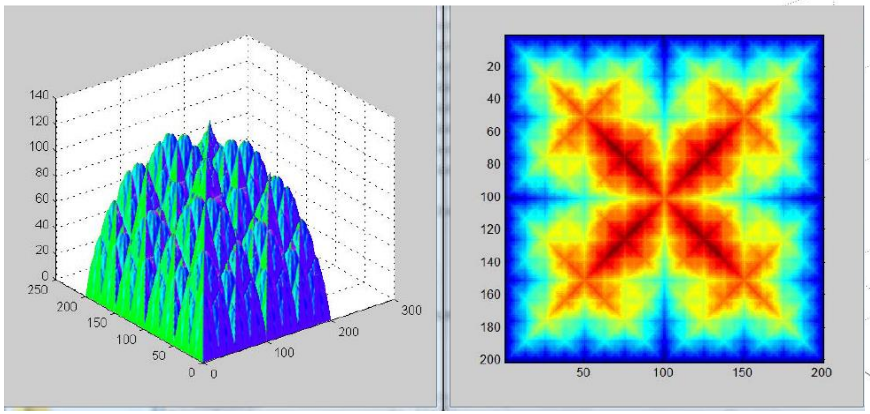


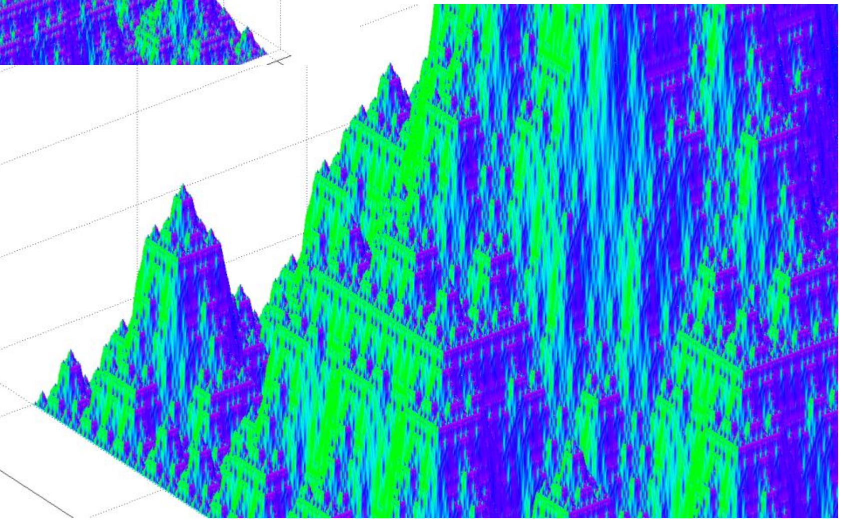
MagicMt(π)



Evolution







MagicMt(0)

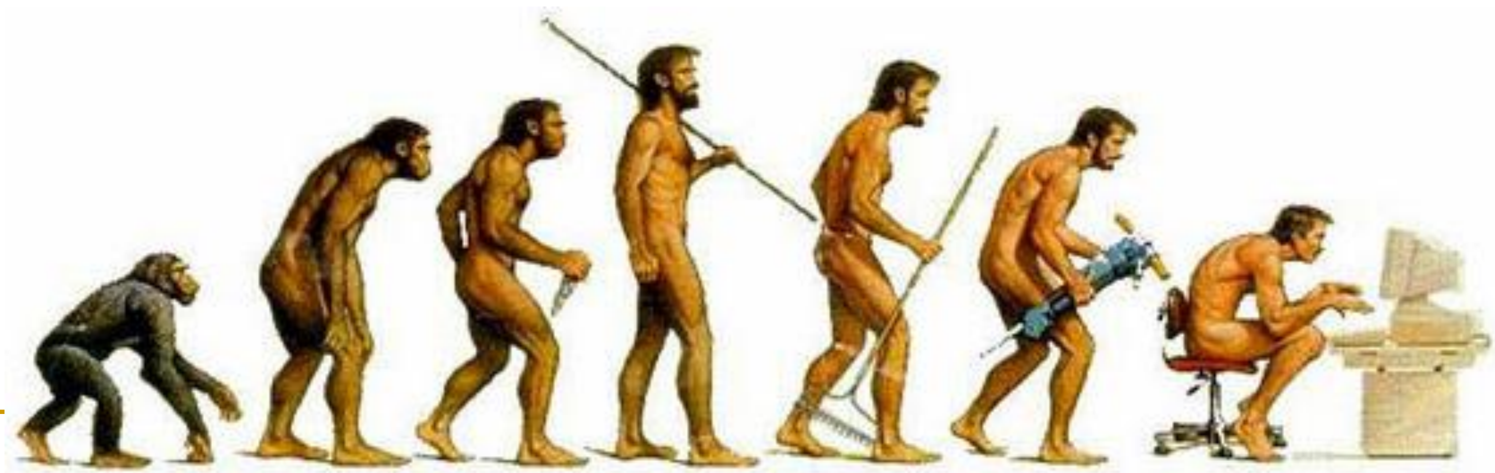
動機:音楽との連携可能性

進化 ＝＞
即座変身（超高速進化）

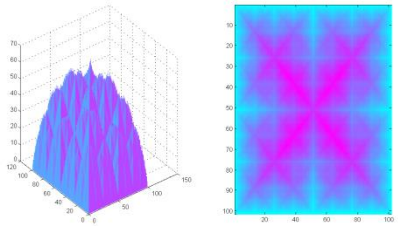
$$\mathrm{MagicMt}(\pi)$$

$$\mathrm{MagicMt}(t) = \sum_{n=0}^{\infty} \cos(nt)\,\mathrm{Pyramid}(n)$$

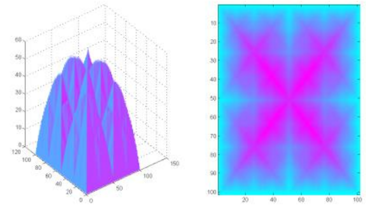
$$t = \frac{0\pi}{30}, \frac{1\pi}{30}, \frac{2\pi}{30}, \frac{3\pi}{30}, \frac{4\pi}{30}, \frac{5\pi}{30}, \ldots, \frac{29\pi}{30}, \frac{30\pi}{30}$$



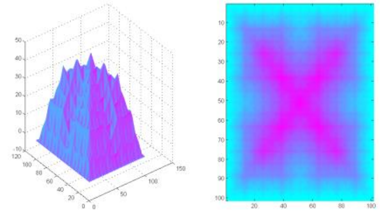
即座変身（新しい見方）



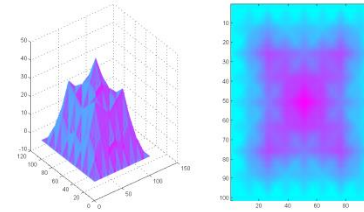
0 microsecond



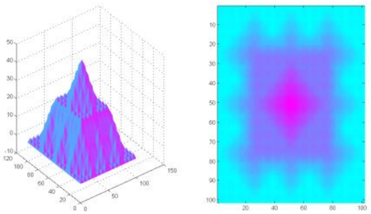
3 microsecond



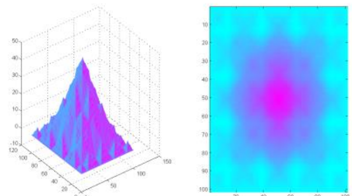
6 microsecond



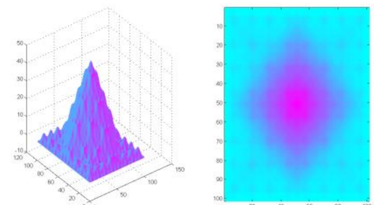
9 microsecond



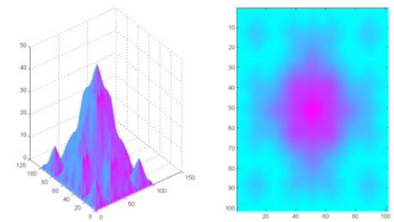
12 microsecond



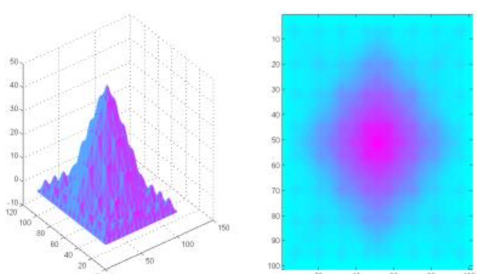
15 microsecond



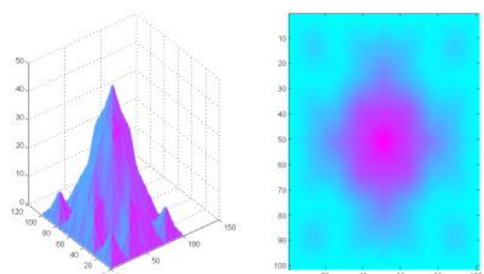
18 microsecond



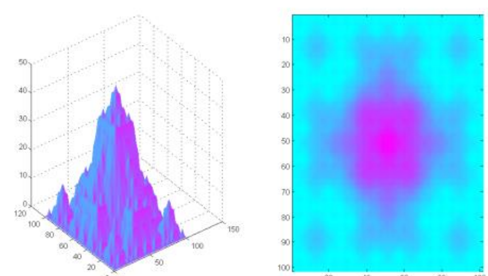
21 microsecond



24 microsecond



27 microsecond



30 microsecond

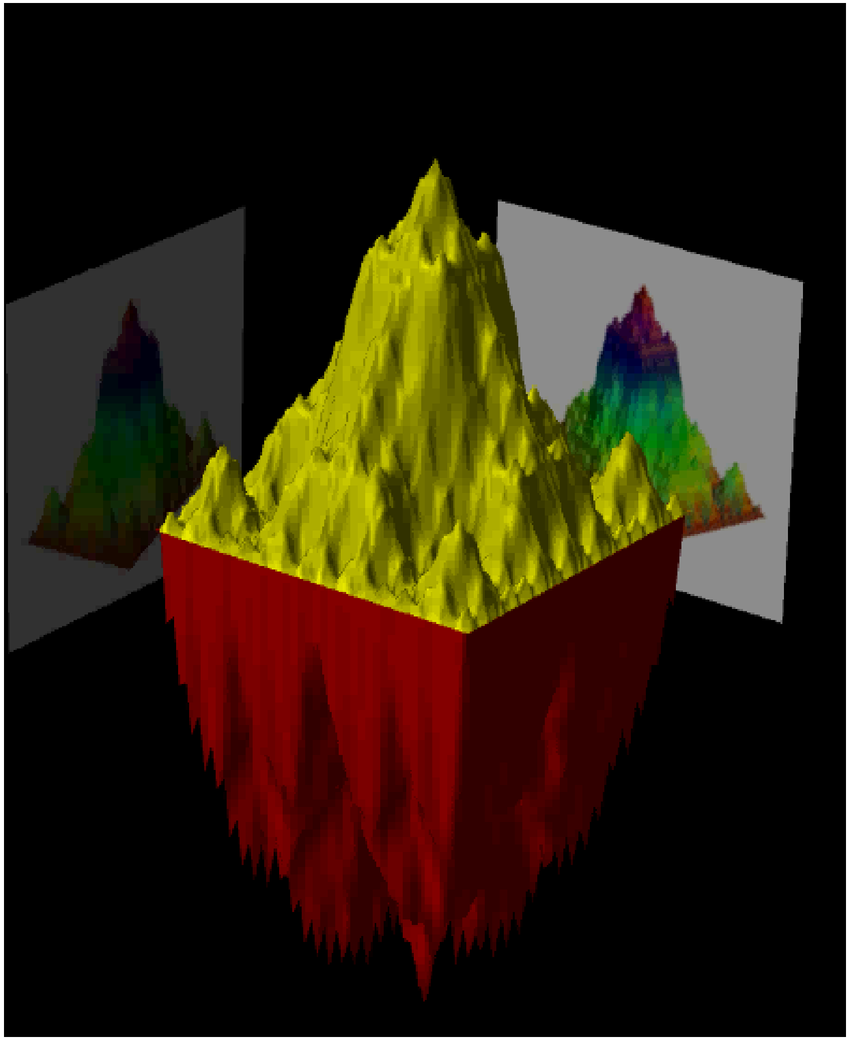
Publications

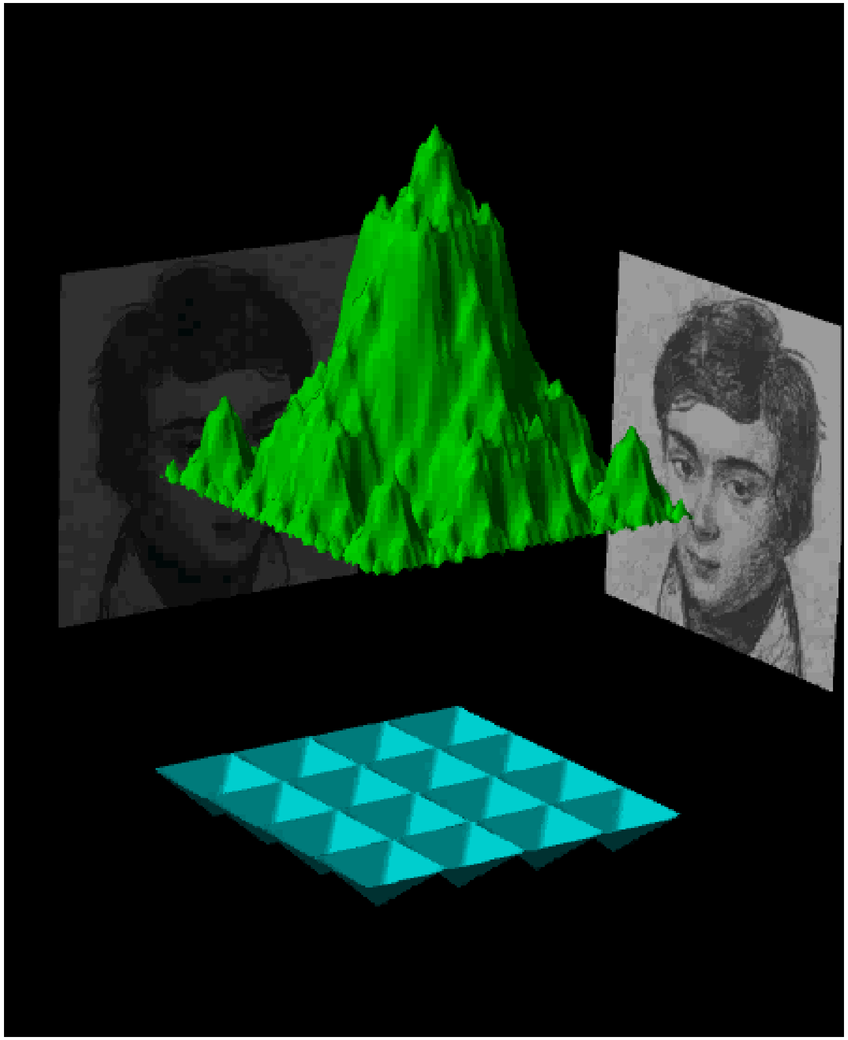
The Matrix Art employed in Tsuyama NCT has recently been published in the international Journal of Mathematical Chemistry, Springer

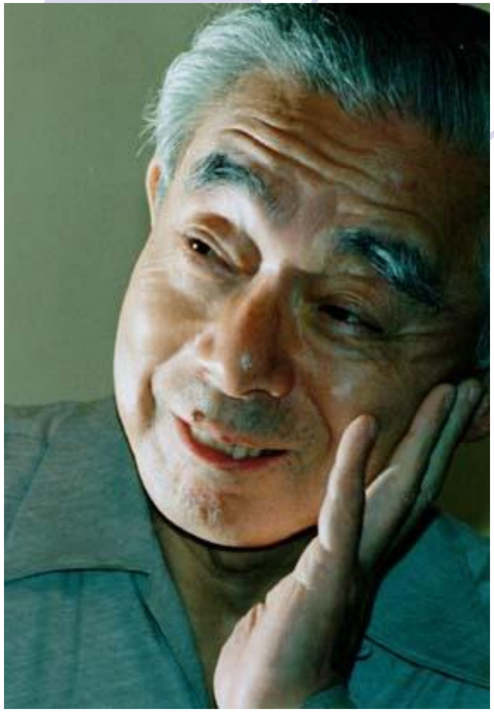
S. Arimoto, Fundamental notions for the second generation Fukui project and a prototypal problem of the normed repeat space and its super spaces, J. Math. Chem. 49 (2011) 880].

S. S. Arimoto, M. Spivakovsky, E. Yoshida, K.F. Taylor, and P.G. Mezey, Proof of the Fukui conjecture via resolution of singularities and related methods. V, J. Math. Chem. 49 (2011) 1700.

S. Arimoto, M. Spivakovsky, M. Amini, E. Yoshida, M. Yokotani, T. Yamabe, Repeat space theory applied to carbon nanotubes and related molecular networks. III, to appear in J. Math. Chem.







Kenichi Fukui (1918 - 1998)

The Fukui Conjecture (Main Part).

Let $\{M_N\}$ be a fixed element of $X_r(q)$ (the repeat space with block-size q), and let I be a fixed closed interval on the real line such that I contains all the eigenvalues of M_N for all positive integers N. Let f: $I \to R$ denote the function defined by

$$f(t) = \frac{\hbar}{2}|t|^{1/2}.$$

Then, there exist real numbers α and β such that

$$E_N(f) = \sum_{i=1}^{qN} f(\lambda_i(M_N)) = \mathrm{Tr}\, f(M_N)$$

$$= \alpha(f)N + \beta(f) + o(1)$$

as $N \to \infty$.

ALT (Asymptotic Linearity Theorem)

$\Rightarrow$ If f is absolutely continuous, then $E_N(f)$ has an asymptotic line.

$\Rightarrow$ **ALT Extension Conjecture (ALTEC) :**

It is impossible to extend the ALT to continuous functions.

ALTEC

Asymptotic Linearity Theorem Extension Conjecture (ALTEC), $C(I)$ version. *The Asymptotic Linearity Theorem* (*ALT*) *can not be extended from* $AC(I)$ *to* $C(I)$, *where* $AC(I)$ *denotes the functional space of all real valued absolutely continuous functions defined on closed interval I, and* $C(I)$ *denotes the functional space of all real valued continuous functions defined on closed interval I.*

[ALTEC1] S. Arimoto, Open problem, Magic Mountain and Devil's Staircase swapping problems, J. Math. Chem. **27** (2000) 213-217.

The initial version of the problems was made public at:

Quantum Physics Centennial Symposium,

March 17-19, 2000 University of Saskatchewan, Canada

ALTEC

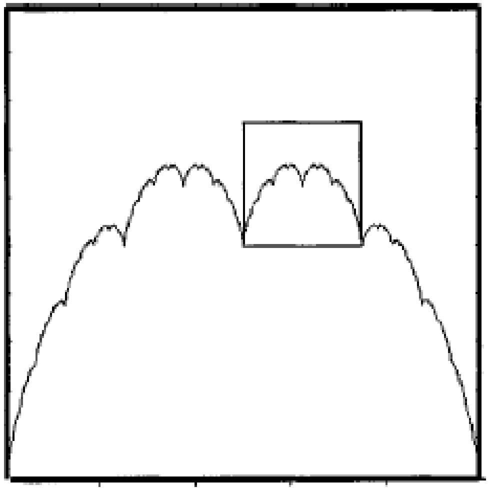


Figure 2. Magic Mountain.

3. Open problems

Now recall the definition of $F_N(\varphi)$ given by equation (5). We know that $(\hbar/2)F_N(\mathcal{R}_{1/2})$ expresses the zero-point energy of the linear chain $Ch_N(4, 1)$ and that $F_N(\mathcal{R}_{1/2})$ has an asymptotic line. What if one swaps $\mathcal{R}_{1/2}$ with Magic Mountain or Devil's Staircase of an arbitrarily given and fixed type η, $0 < \eta \leqslant 1$? Here, then, are our two open problems:

(I) **Magic Mountain swapping problem.**
Does the sequence $F_N(\mathcal{M})$ have an asymptotic line?

(II) **Devil's Staircase swapping problem.**
Does the sequence $F_N(\mathcal{D}_\eta)$ have an asymptotic line?

Remark. If n is an arbitrarily given and fixed positive integer, then $\mathcal{M}_n$ is absolutely continuous on the closed interval $[0, 1]$, and the Asymptotic Linearity Theorem (ALT) implies that the sequence $F_N(\mathcal{M}_n)$ has an asymptotic line.

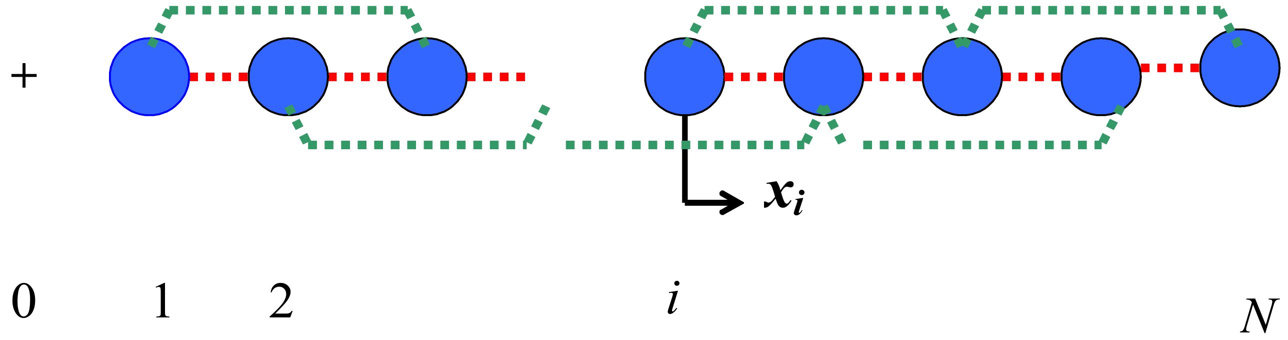
Genesis of the Approach via the Aspect of Form and General Topology

Linear Chain $Ch_N(m, k_1, k_2)$

Mass of each particle: m

The 1st nbd force constant: k_1

The 2nd nbd force constant: k_2



Hamiltonian Operator $H_N(m, k_1, k_2)$ and Mass-weighted Hessian matrix $M_N(m, k_1, k_2)$ for Linear Chain $Ch_N(m, k_1, k_2)$

$$H_N(m,k_1,k_2) = \sum_{i=1}^{N} -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + \frac{m}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(M_N(m,k_1,k_2))_{ij}\, x_i x_j,$$

where $M_N(m, k_1, k_2)$ denotes an $N \times N$ matrix given by

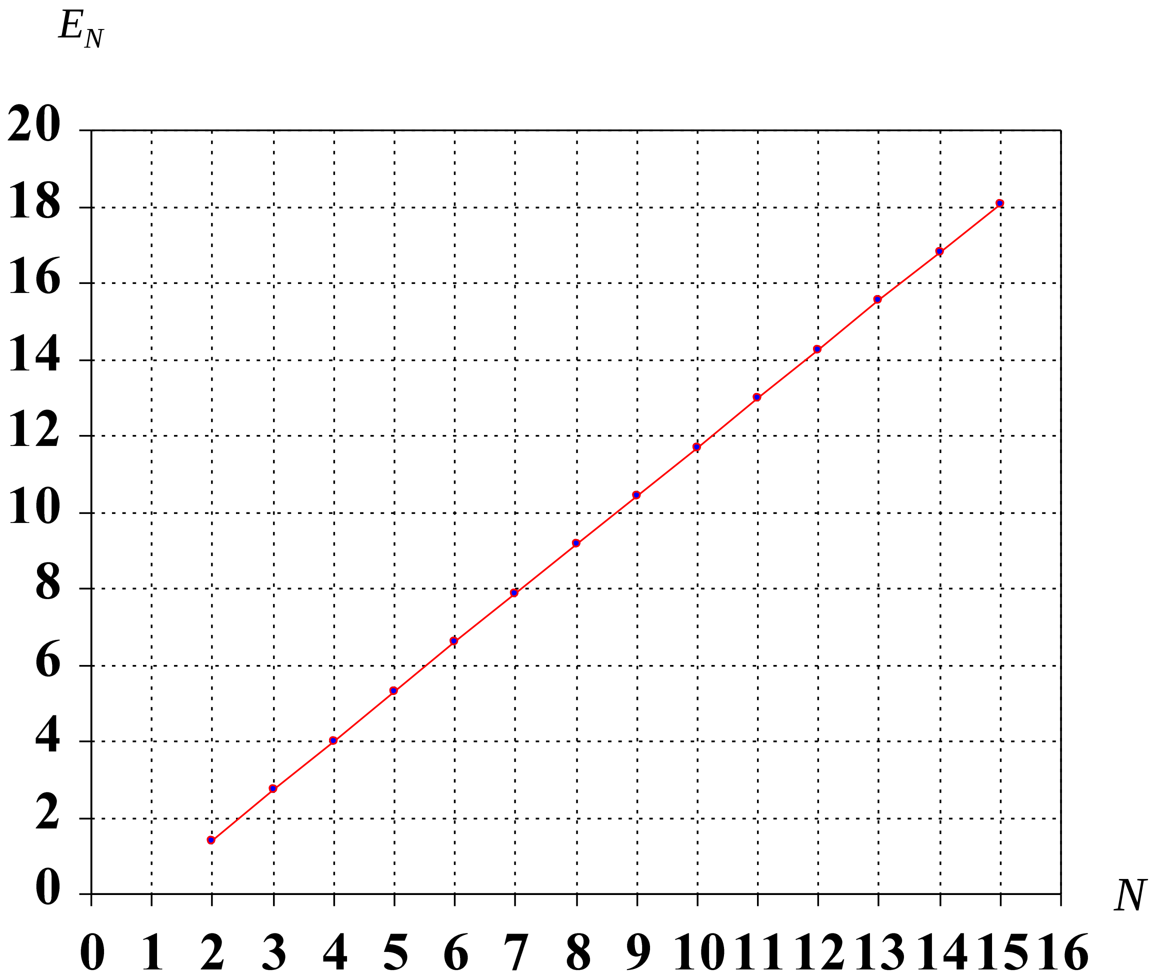
$$M_N(m,k_1,k_2) = \left(\frac{1}{m}\right)\begin{pmatrix} k_1+k_2 & -k_1 & -k_2 & & & & & zeros \\ -k_1 & 2k_1+k_2 & -k_1 & -k_2 & & & & \\ -k_2 & -k_1 & 2(k_1+k_2) & -k_1 & \bullet & & & \\ & -k_2 & -k_1 & 2(k_1+k_2) & \bullet & \bullet & & \\ & & -k_2 & -k_1 & \bullet & \bullet & -k_2 & \\ & & & \bullet & \bullet & 2(k_1+k_2) & -k_1 & -k_2 \\ & & & & \bullet & -k_1 & 2k_1+k_2 & -k_1 \\ zeros & & & & & -k_2 & -k_1 & k_1+k_2 \end{pmatrix}.$$

The Zero-point Energy of $E_N(m, k_1, k_2)$ of Linear Chain $Ch_N(m, k_1, k_2)$

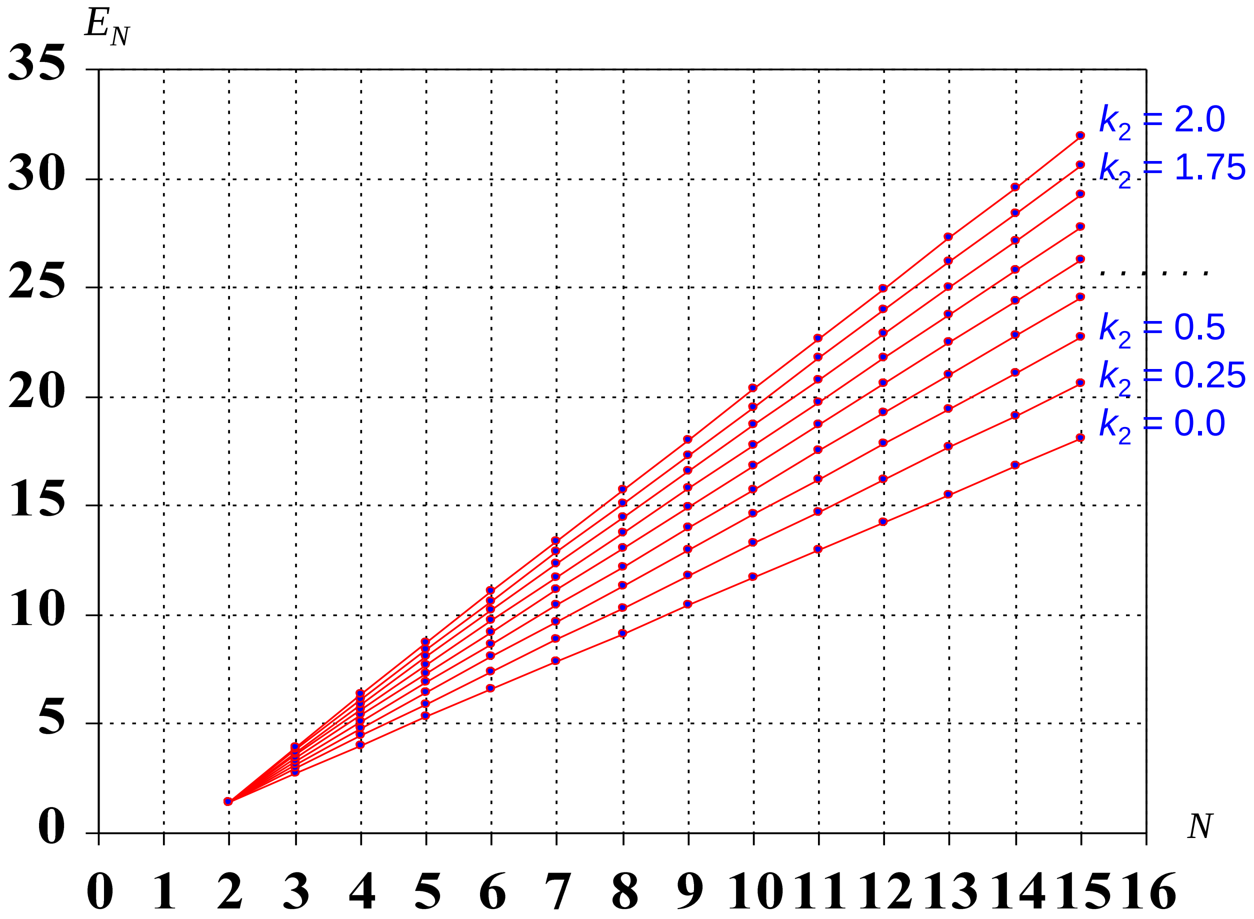
The zero-point energy $E_N(m, k_1, k_2)$ of linear chain $Ch_N(m, k_1, k_2)$ is expressed in terms of the eigenvalues of the mass-weighted Hessian matrix $M_N(m, k_1, k_2)$.

$$E_N(m, k_1, k_2) = \sum_{i=1}^{N} \frac{\hbar}{2} [\lambda_i (M_N(m, k_1, k_2))]^{1/2}$$

Zero-point Energy E_N of Linear Chain $Ch_N(1, 1, 0)$



Zero-point Energy E_N of Linear Chain $Ch_N(1, 1, k_2)$



Approach via Difference Equations

(1) The eigenvalues of $M_N(m, k, 0)$ are analytically obtainable:

$$\lambda_i(M_N(m, k, 0)) = 4((k/m)\sin^2[(i -1)\pi/(2N)].$$

Difference Eq.

$$f(n + 2) + (\lambda - 2)f(n + 1) + f(n) = 0, \quad (n = 1, \ldots, N)$$

$$f(1)(1 - \lambda) - f(2) = -f(n - 1) + (1 - \lambda)f(n) = 0.$$

(**Boundary Condition**)

(2) The zero-point energy $E_N(m, k)$ of Linear Chain $\mathrm{Ch}_N(m, k, 0)$ is obtainable by applying a simple formula for the sum of the trigonometric functions. **[$E_N(m, k)$ has an asymptotic line!]**

$$E_N(m,k) = \frac{\hbar}{2}\sqrt{\frac{k}{m}}[\cot\left(\frac{\pi}{4N}\right) - 1] = \frac{\hbar}{2}\sqrt{\frac{k}{m}}\left(\frac{4N}{\pi} - 1\right) + o(1), N \to \infty.$$

“Instead of the Planck constant, let me change the square root function in the formula for the zero-point energy E_{zero} of a linear oscillator!”

$$E_{zero} = \frac{\hbar}{2}\left(\frac{k}{m}\right)^{1/2}.$$

$$E_{zero} = \frac{\hbar}{2}\left(\frac{k}{m}\right)^{u}.$$

Some Change of a Statement in FORTRAN Code

```
00124 SIG = 0.0
00125 DO 150 I = 1,K
00126 SIG = SIG + SQRT(A(I,I))
00127 150 CONTINUE
```

```
00124 SIG = 0.0
00125 DO 150 I = 1,K
00126 SIG = SIG + (A(I,I))**U
00127 150 CONTINUE
```

The Planck Constant as a Variable

Special magnitudes of universal constants and specific forms of functions manifest themselves in the expressions of natural laws. Nevertheless, it is sometimes legitimate and meaningful to embed fixed constants or functions into a broader context and make them change. For example, one can regard the Planck constant as a variable and let it tend to zero so that classical mechanics can be considered as a limit of quantum mechanics.

George Gamow reversed this process and magnified the Planck constant in his instructive and amusing book *Mr. Tompkins in Wonderland*. In Gamow's book, Mr. Tompkins, the little clerk of a big city bank experiences a manifestation of Heisenberg's uncertainty principle in a billiard room filled with men in shirt sleeves playing billiards:

Functions $f_0,\ f_T, u_T, g_{\xi} \ \in AC(I)$

$$E_0 = \underset{j}{\bullet}\ f_0(l_j), \quad \text{where} \quad f_0(x) = \frac{\hbar |x|^{1/2}}{2}.$$

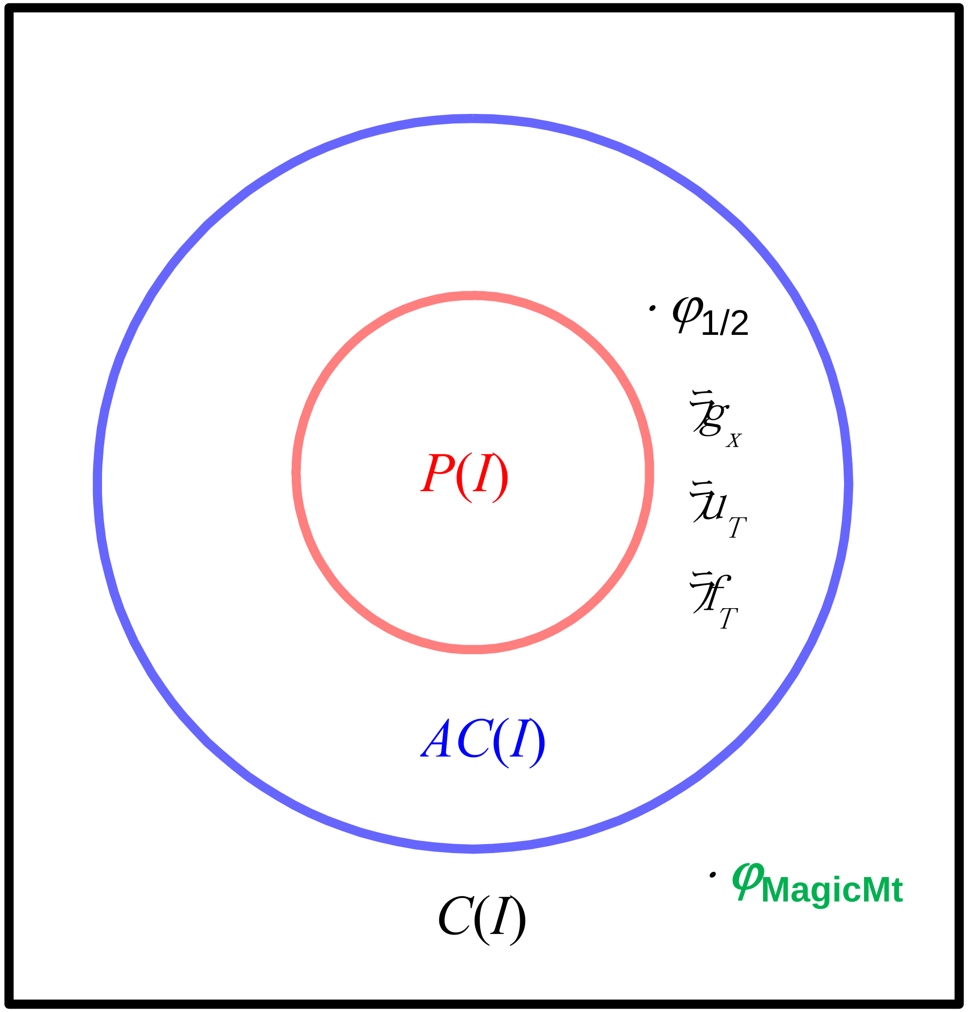
$$E_T = \underset{j}{\bullet}\ f_T(l_j), \quad \text{where} \quad f_T(x) = \frac{\hbar |x|^{1/2}}{2} \coth\left(\frac{\hbar |x|^{1/2}}{2kT} \right).$$

$$C_V = \underset{j}{\bullet}\ u_T(l_j), \quad \text{where} \quad u_T(x) = k\left(\frac{\hbar |x|^{1/2}}{2} \right)^2 / \sinh^2\left(\frac{\hbar |x|^{1/2}}{2kT} \right).$$

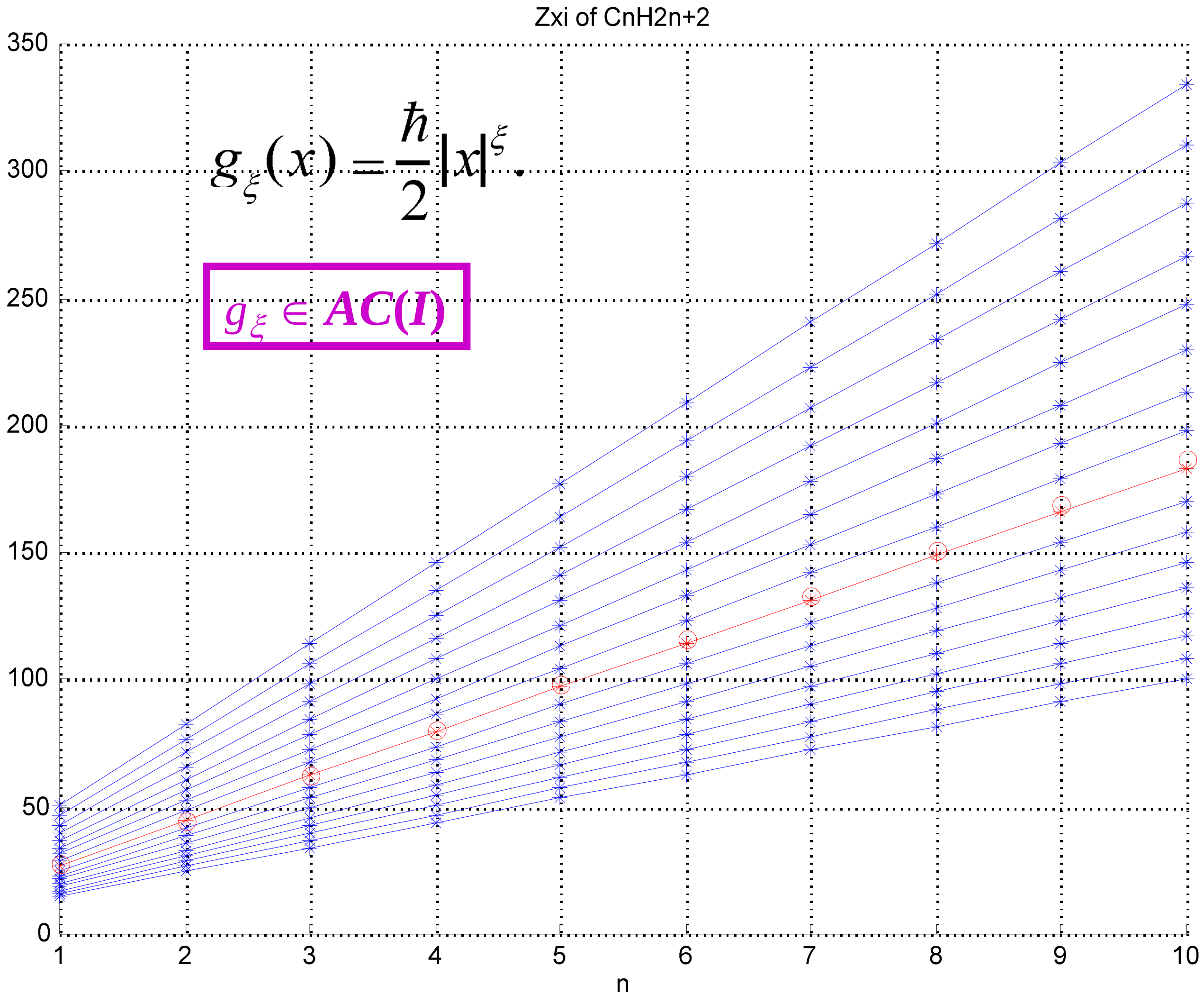
$$E_{TPEE-type} = \underset{j}{\bullet}\ g_x(l_j), \quad \text{where} \quad g_{\xi}(x) = const \cdot |x|^{\xi}.$$

$$* = \underset{j}{\bullet}\ function(l_j), \quad \text{where} \quad l_j = jth - Eigenvalue(M_N).$$

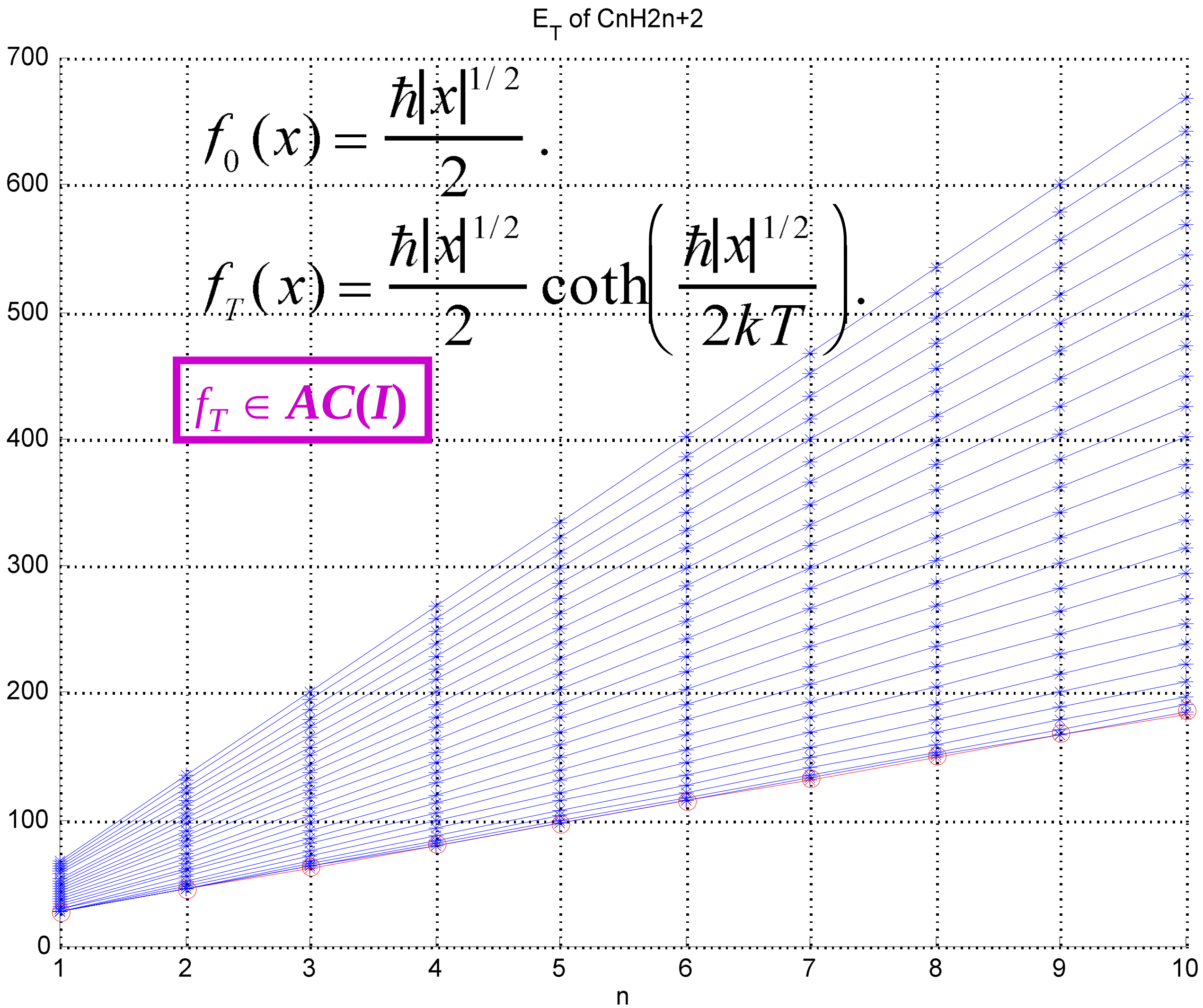
The Banach Space $AC(I)$



Z_ξ Plotting of $C_n H_{2n+2}$ $(0.46 \le \xi \le 0.54)$



Internal Energy of $C_n H_{2n+2}$ as $T \to 0$ (ºK)

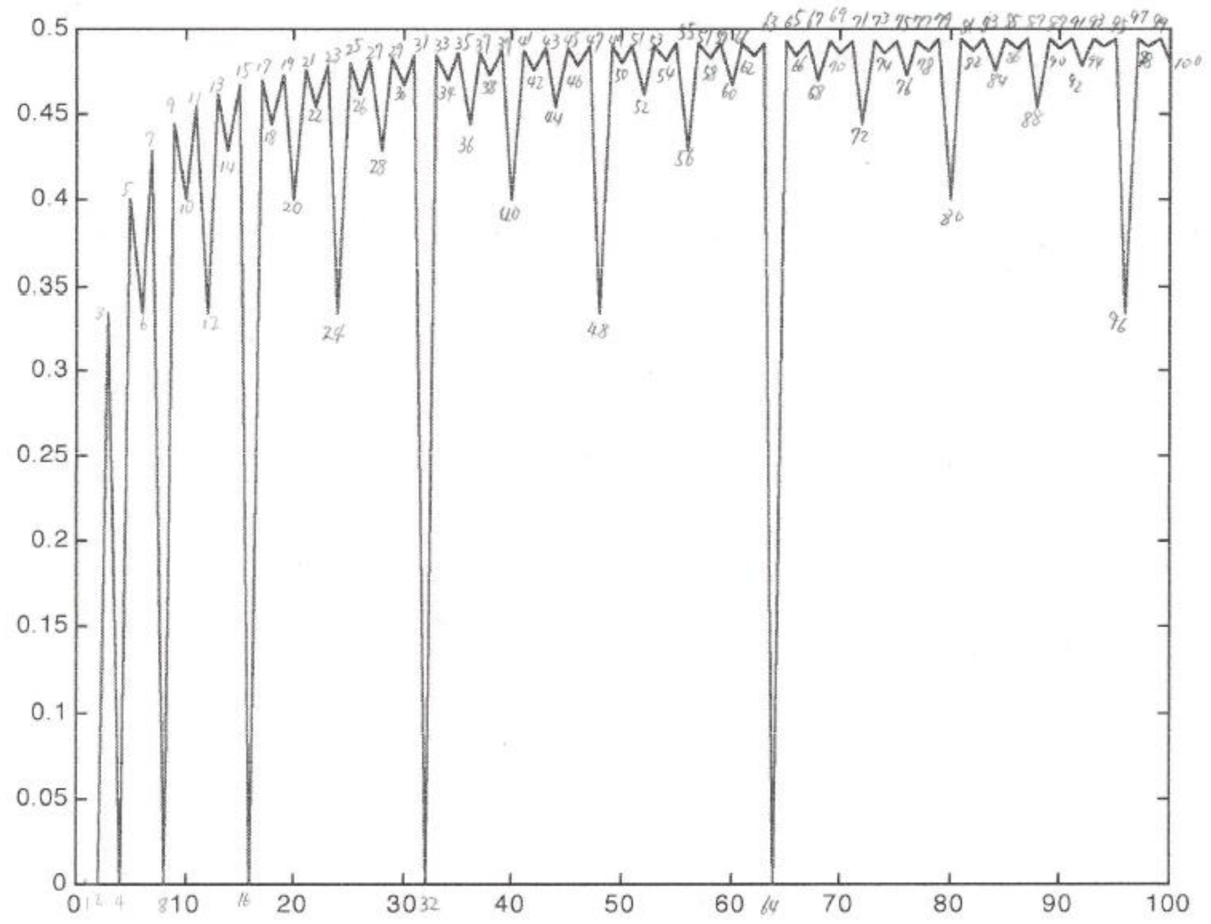


$$f_0(x) = \frac{\hbar |x|^{1/2}}{2}.$$

$$f_T(x) = \frac{\hbar |x|^{1/2}}{2} \coth\left(\frac{\hbar |x|^{1/2}}{2kT}\right).$$

$f_T \in AC(I)$

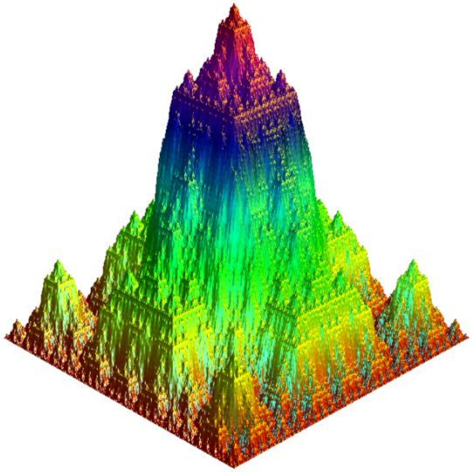
Numerical Evidence for ALTEC



Science-Art Multi-angle Network 計画

福井プロジェクト・グループ主催の上記計画はグローバル化の現代において科学と芸術（視覚的芸術，聴覚的芸術，文学）と哲学とを結び，個性的特徴をもった研究・教育機関，及び美術館等が各々の特徴を生かしつつ相互補足的にSkypeやEvernote 等のクラウド・ソフトを用いつつ（岡山県や日本をも超えて）広域的に連携しあう，「公益性と地域活性」の両方のメリットをもったプログラムであります。（詳細は次の短縮URL中のデータを参照：http://bit.ly/1Mrbd2R）

Tsuyama Kousen Special Version（Technoセンター報）:分野横断力をそなえたグローバル人材の育成は，高専のみならず初等教育から高等教育にわたる教育機関全般における現代の重要な課題の一つであります。本研究室では，津山高専紀要第55号（2013）pp.25-43，第56号（2014）pp. 17-53の論文執筆者の協力を得ながら，津山高専所有のインターネット会議システム，Evernote，Skype，Desmos等のクラウドを利用して，（I）講演録（II）文理横断的，研究，教育データベース（III）学生用，クラウド教材データベース構築のプロジェクトを開始しております。



ALTEC

Science-Art Multi-angle Network

: http://bit.ly/1Mrbd2R

Thank you!

御清聴有難うございました。

有本　茂「<数学と諸科学＞福井予想とNew Frontier Project　学際研究」数学、日本数学会（２０１５年出版確定）

山口昌哉先生の著書「カオスとフラクタル」(ちくま学芸文庫, 1986年)のはしがきには「今となってみれば, 日本人こそ, もう少し広い視野と交流があればできたことではないかと思う. 残念ながら, 現在の日本は流行を追うことに忙しく, 世界に流行を作り出すことは, 結局ほとんどだれ一人関心がないのではないかと思われる」という示唆に富む文章がある. 筆者が1990年カナダに渡航する数日前, その準備で家はカオス状態であったが, 山口先生の義理の妹さんヴァレン文子さんの御好意で, 京都のあるフランス料理店に行けば山口先生とお会いできるだろうとの機会が与えられた. しかし, 自転車ででかけた筆者の方向誤認とその夜の小雨交じりの台風でお会いするチャンスを逃がしてしまったのは真に残念である. フラクタル関数等の凸凹の激しい関数が低速漸近性や振動発散をもたらし, その対極にある福井予想や高速漸近性の研究に役立つと気づいたのはカナダに行って何年もたった後の事であったが, あの日, びしょ濡れになっていても正しい方向と場所を見出し, もし, お会いしておればと後悔の念が絶えない.